

# Heuristic Search

**CSE-345: Artificial Intelligence**

# Heuristic Search

The search techniques we have seen so far...

- Breadth first search
- Uniform cost search
- Depth first search
- Depth limited search
- Iterative Deepening
- Bi-directional Search

← uninformed search  
blind search

...are all too slow for most real world problems

# 8-Puzzle

Solution in Node 46

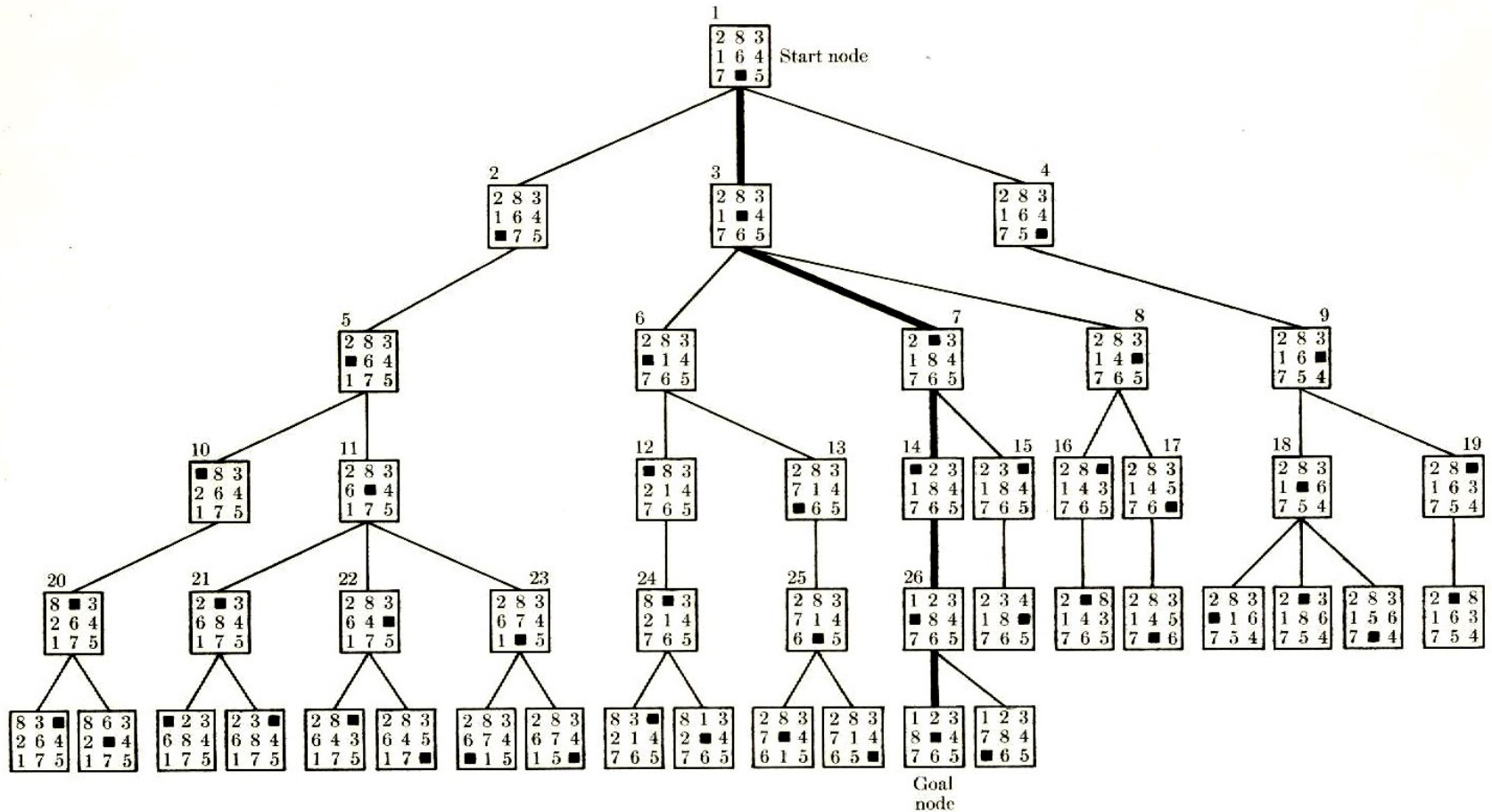


FIG. 3-2 The tree produced by a breadth-first search.

# 8-Puzzle

Solution in Node 31

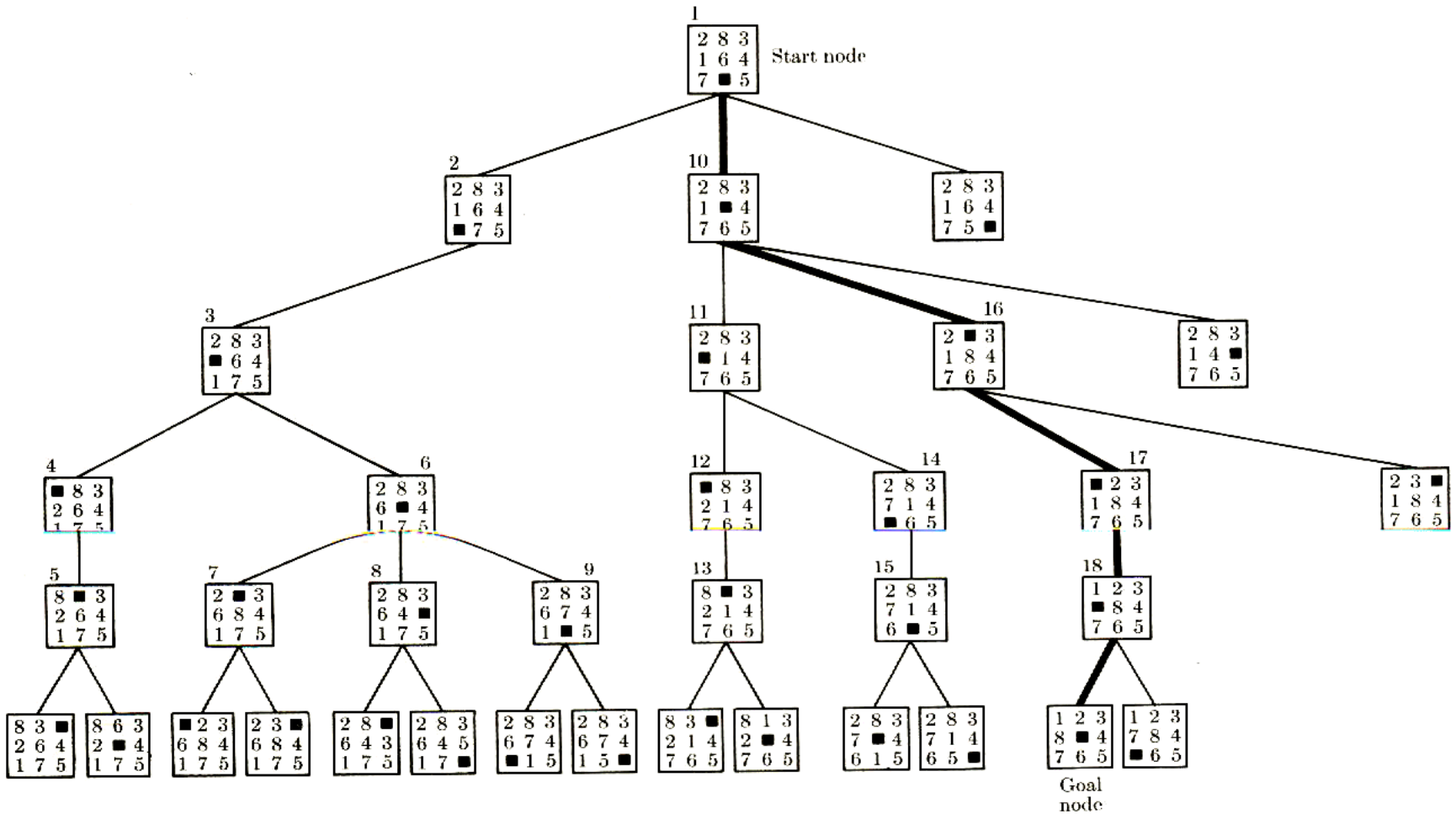
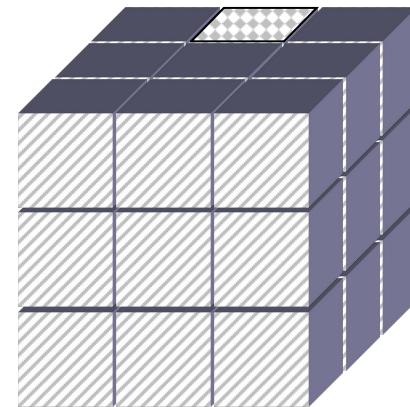
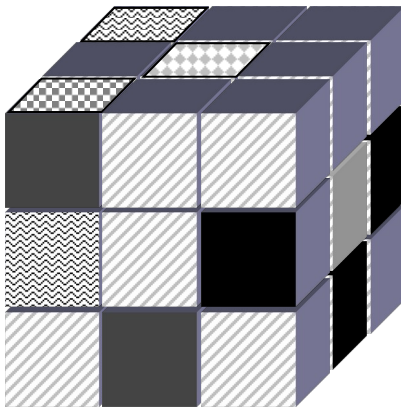


FIG. 3-5 The tree produced by a depth-first search.

Sometimes we can tell that some states appear better than others...

7	8	4
3	5	1
6	2	

1	2	3
4	5	6
7		8



..we can use this knowledge of the relative merit of states to guide search

## Heuristic Search (informed search)

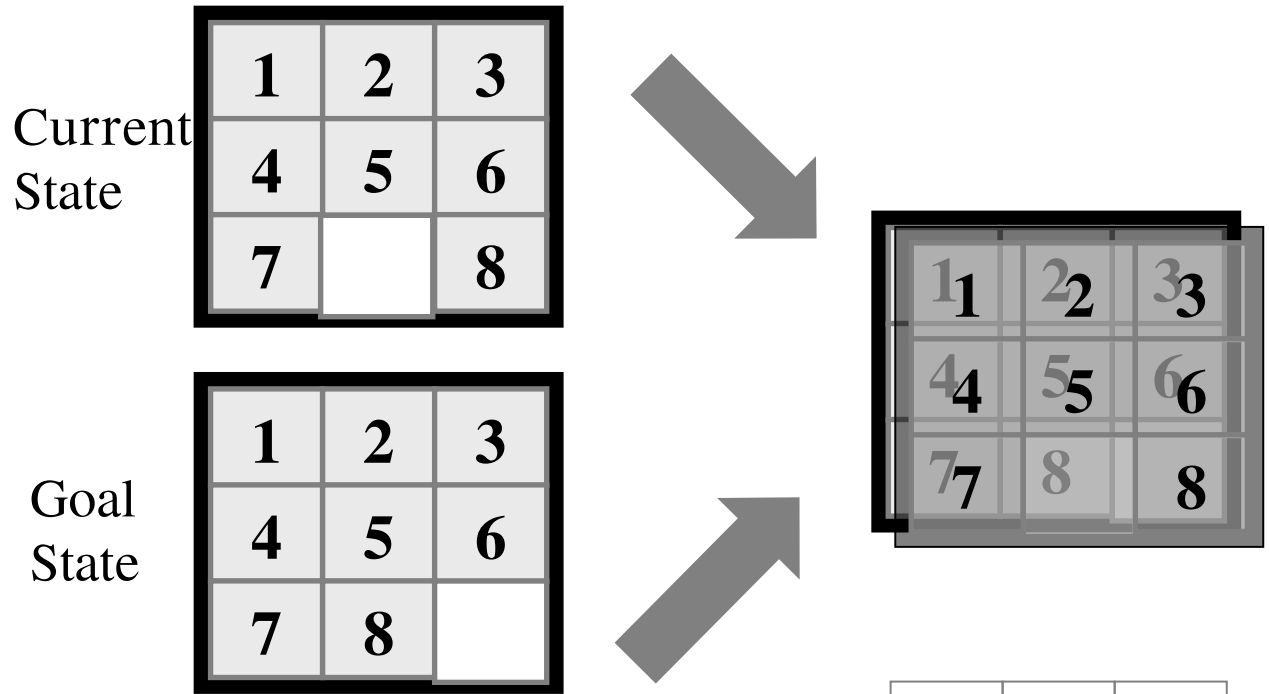
A **Heuristic** is a function that, when applied to a state, returns a number that is an estimate of the merit of the state, with respect to the goal.

In other words, the heuristic tells us approximately how far the state is from the goal state\*.

Note we said “approximately”. Heuristics might underestimate or overestimate the merit of a state. But for reasons which we will see, heuristics that *only* underestimate are very desirable, and are called admissible.

\*I.e Smaller numbers are better

# Heuristics for 8-puzzle I



- The number of **misplaced tiles** (not including the blank)

In this case, only “8” is misplaced, so the heuristic function evaluates to 1.

In other words, the heuristic is *telling us*, that it *thinks* a solution might be available in just 1 more move.

N	N	N
N	N	N
N	Y	

Notation:  $h(n)$        $h(\text{current state}) = 1$

# Heuristics for 8-puzzle II

Current State

3	2	8
4	5	6
7	1	

•The Manhattan Distance (not including the blank)

Goal State

1	2	3
4	5	6
7	8	

3	→	<u>3</u>

2 spaces

	←	8
	↓	
	<u>8</u>	

3 spaces

<u>1</u>	←	
	↑	
	1	

3 spaces

Total 8

In this case, only the “3”, “8” and “1” tiles are misplaced, by 2, 3, and 3 squares respectively, so the heuristic function evaluates to 8.

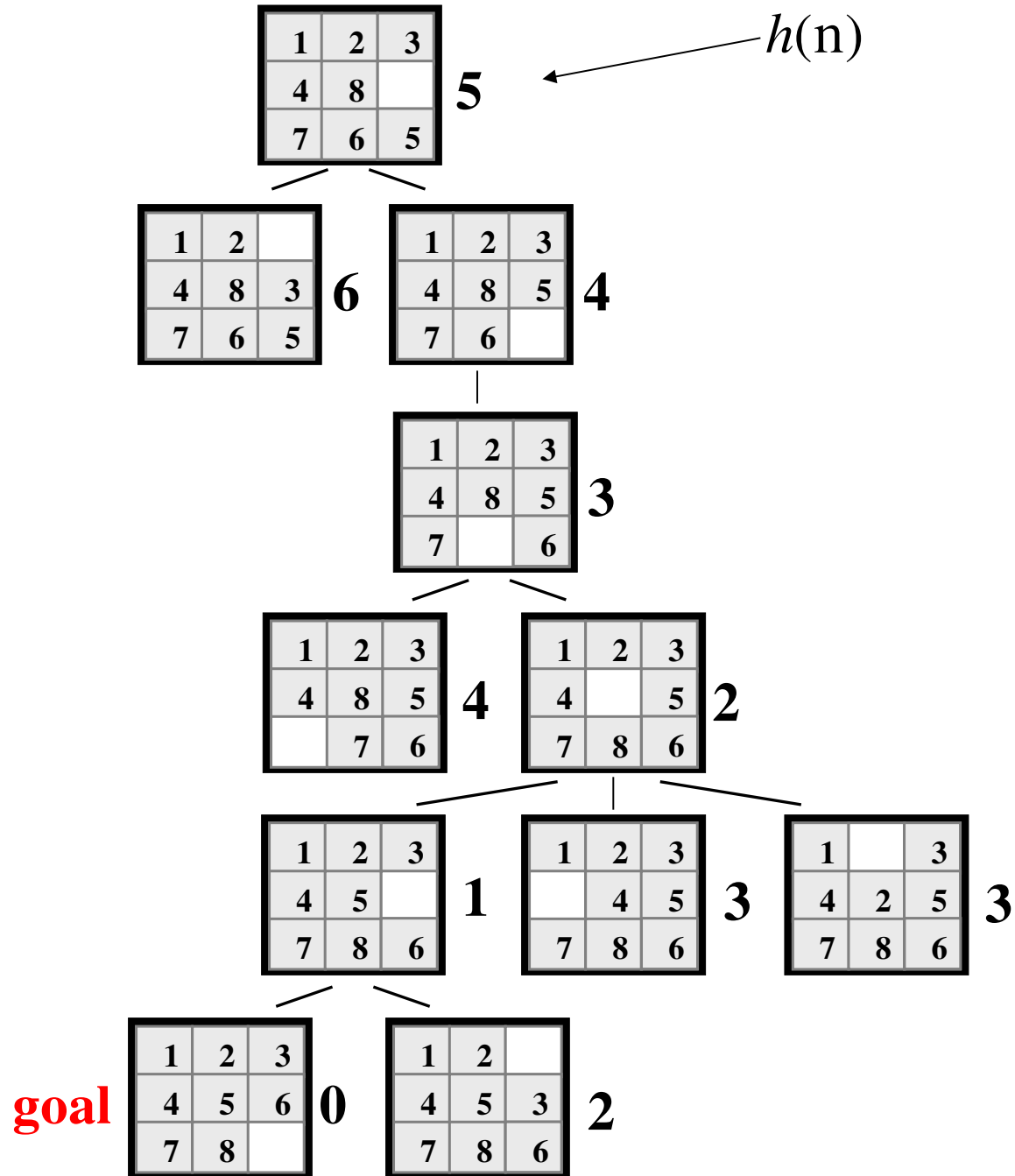
In other words, the heuristic is *telling* us, that it *thinks* a solution is available in just 8 more moves.

Notation:  $h(n)$

$h(\text{current state}) = 8$

We can use heuristics to guide search.

In this example, the Manhattan Distance heuristic helps us quickly find a solution to the 8-puzzle.



# Heuristic Search

## □ **Heuristic**

- **heuriskein (Greek word)**-to discover
- **original: eureka**  
-heurika (I have found): Archimedes uttering when determining the purity of gold

## □ **Heuristic Function**

- a way to inform the search about the direction to a goal.
- provides an informed way to guess which neighbor of a node will lead to a goal.

## □ **Pros**

- Like a tour guide
- Good to the extent that they point in generally interesting direction
- Improve the quality of the paths that are explored
- Using good heuristic, we can hope to get good solutions to hard problem
- It is possible to construct special-purpose heuristics that exploit domain-specific knowledge to solve particular problems

# Heuristic Search

- A *heuristic* is a method that
  - might not always find the best solution
  - but** is guaranteed to find a good solution in reasonable time.
- ✓ By sacrificing completeness it increases efficiency.
- ✓ Useful in solving **tough problems** which
  - could not be solved any other way.
  - solutions take an infinite time or very long time to compute.
- The *classic* example of heuristic search methods is the **travelling salesman problem**.

# Heuristic Search

## □ **Traveling Salesman Problem**

1. Arbitrarily select a starting city
2. To select the next city, look at all cities not yet visited,
  - Select the one closest to the current city.
  - Go to it next
3. Repeat step 2 until all cities have been visited

# Heuristic Function

- A *heuristic function* is a function that maps from problem state descriptions to measure of desirability, usually represented as numbers
  - Which aspects of the problem state are considered?
  - How those aspects are evaluated?
  - The weights given to individual aspects are chosen in such a way that the value of the heuristic function at a given node in the search process gives as good an estimate as possible of whether that node is on the desired path to a solution
- Well-designed heuristic functions can play an important part in efficiently guiding a search process toward a solution
- The purpose of a heuristic function is to guide the search process in the most profitable direction by suggesting which path to follow first when more than one is available
- The more accurately the heuristic function estimates the true merits of each node in the search tree/graph, the more direct the solution process

# Heuristic Search

- Generate –and- test
- **Hill Climbing**
  - Simple Hill Climbing
  - Steepest-Ascent Hill Climbing
  - **Simulated Annealing**
- **Best-First Search**
  - OR Graphs
  - **A\* Algorithm**
  - **Greedy Best-First Search**
- **Problem Reduction**
  - **AND-OR Graphs**
  - AO\* Algorithm
- **Constraint Satisfaction**
- Means-ends Analysis

# Generate-and-Test

## Algorithm

1. Generate a possible solution.
  2. Test to see if this is actually a solution.
  3. Quit if a solution has been found.  
Otherwise, return to step 1.
- ✓ Acceptable for simple problems.
  - ✗ Inefficient for problems with large space.

# Hill Climbing

- A variant of generate-and-test in which feedback from the test procedure is used to help the generator decide which direction to move in the search space.
- Searching for a **goal state** = Climbing to the **top of a hill**
- Generate-and-test + **direction to move**.
- **Heuristic function** to estimate how close a given state is to a goal state.

# Simple Hill Climbing

## Algorithm

1. Evaluate the initial state.
  2. Loop until a solution is found or there are no new operators left to be applied:
    - Select and apply a new operator
- Evaluate the new state:
- goal → quit
  - better than current state → new current state

# Steepest-Ascent Hill Climbing (Gradient Search)

- Considers **all the moves** from the current state.
- Selects **the best one** as the next state.

# Steepest-Ascent Hill Climbing (Gradient Search)

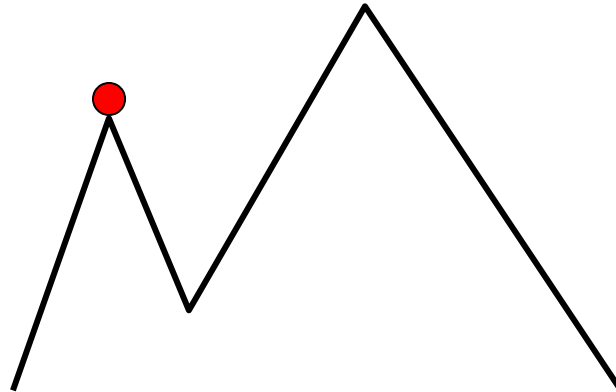
## Algorithm

1. Evaluate the initial state.
  2. Loop until a solution is found or a complete iteration produces no change to current state:
    - SUCC = a state such that any possible successor of the current state will be better than SUCC (the worst state).
- For each operator that applies to the current state, evaluate the new state:
- goal → quit
  - better than SUCC → set SUCC to this state
- SUCC is better than the current state → set the current state to SUCC.

# Hill Climbing: Disadvantages

## Local maximum

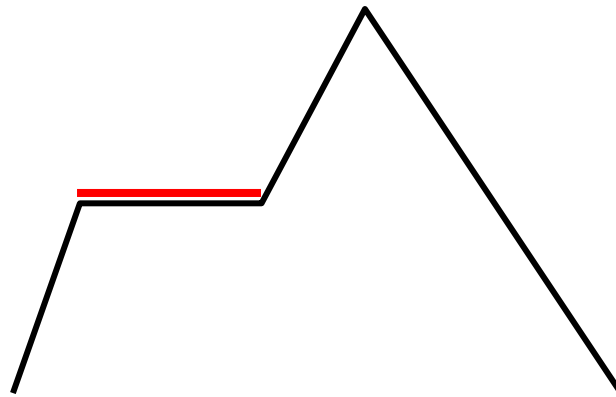
- A state that is better than all of its neighbours, but not better than some other states far away.



# Hill Climbing: Disadvantages

## Plateau

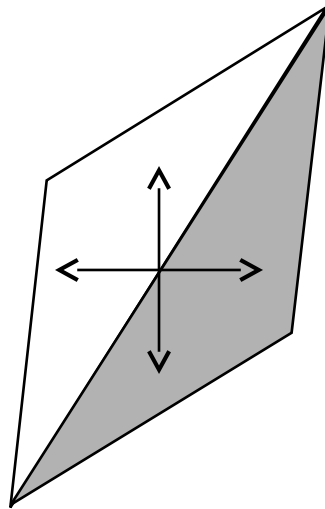
- A flat area of the search space in which all neighbouring states have the same value.



# Hill Climbing: Disadvantages

## Ridge

➤ The orientation of the high region, compared to the set of available moves, makes it impossible to climb up. However, two moves executed serially may increase the height.



# Hill Climbing: Disadvantages

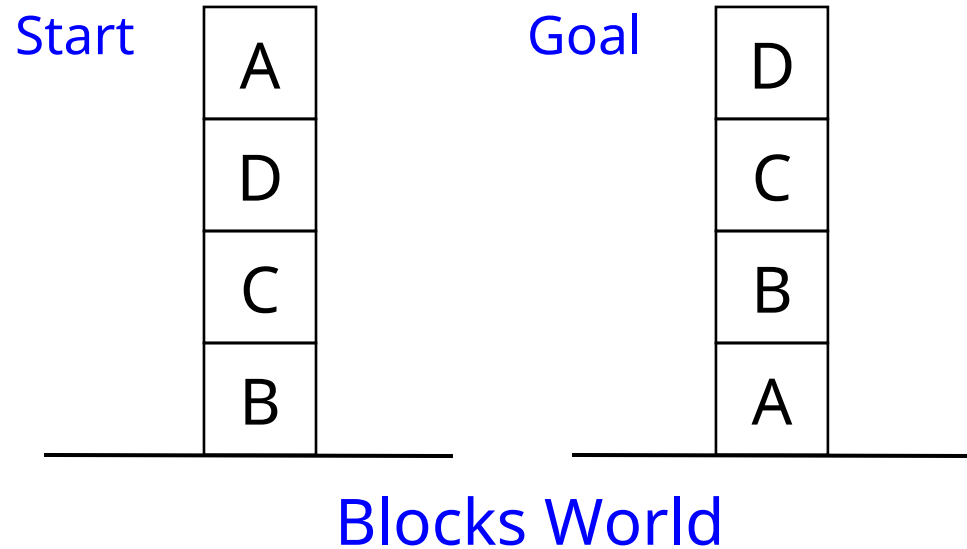
## Ways Out

- **Backtrack** to some earlier node and try going in a different direction.
- Make a **big jump** to try to get in a new section.
- Moving in **several directions** at once.

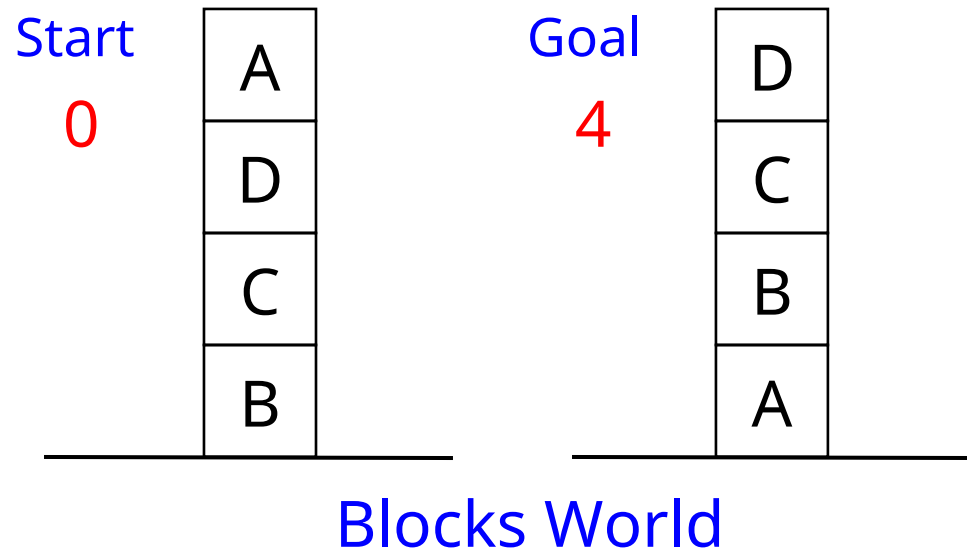
# Hill Climbing: Disadvantages

- Hill climbing is a **local method**:  
Decides what to do next by looking only at the “immediate” consequences of its choices.
- **Global information** might be encoded in heuristic functions.

# Hill Climbing: Disadvantages



# Hill Climbing: Disadvantages

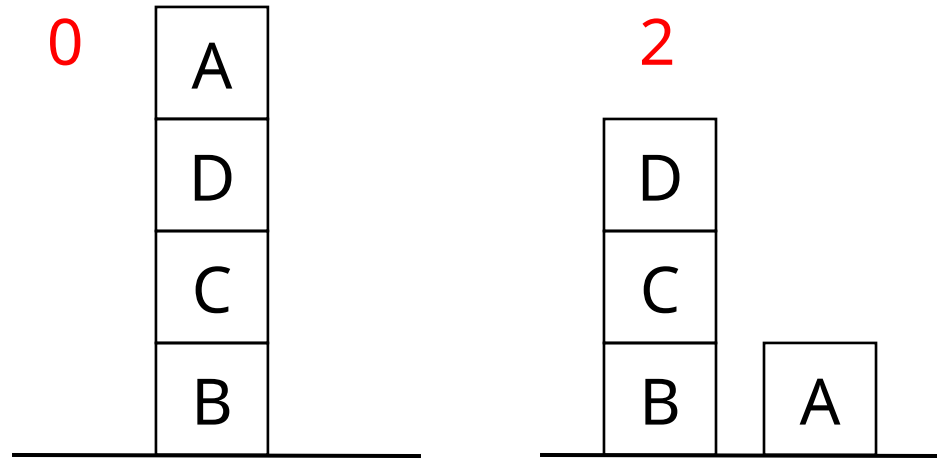


Local heuristic:

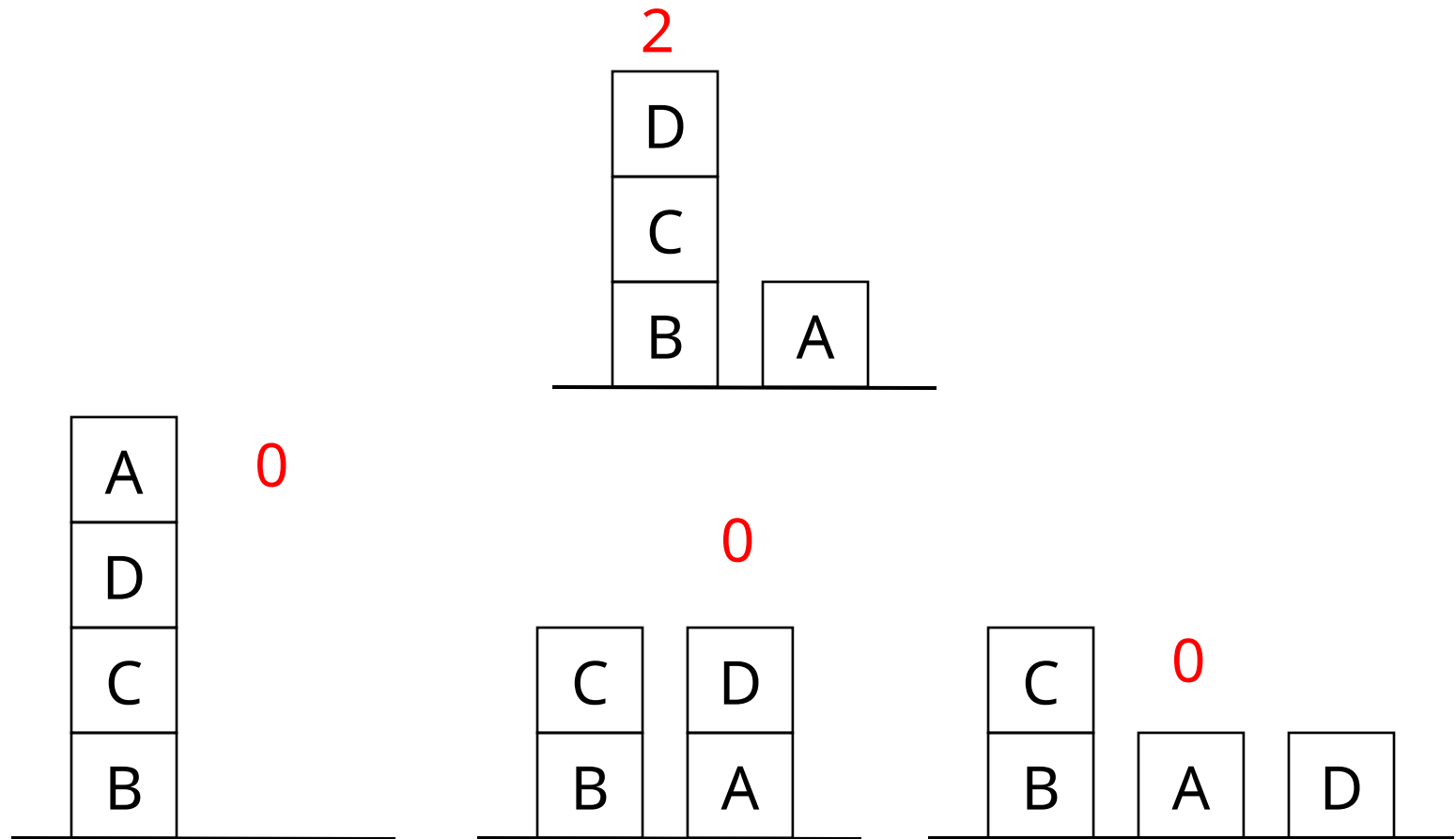
+1 for each block that is resting on the thing it is supposed to be resting on.

-1 for each block that is resting on a wrong thing.

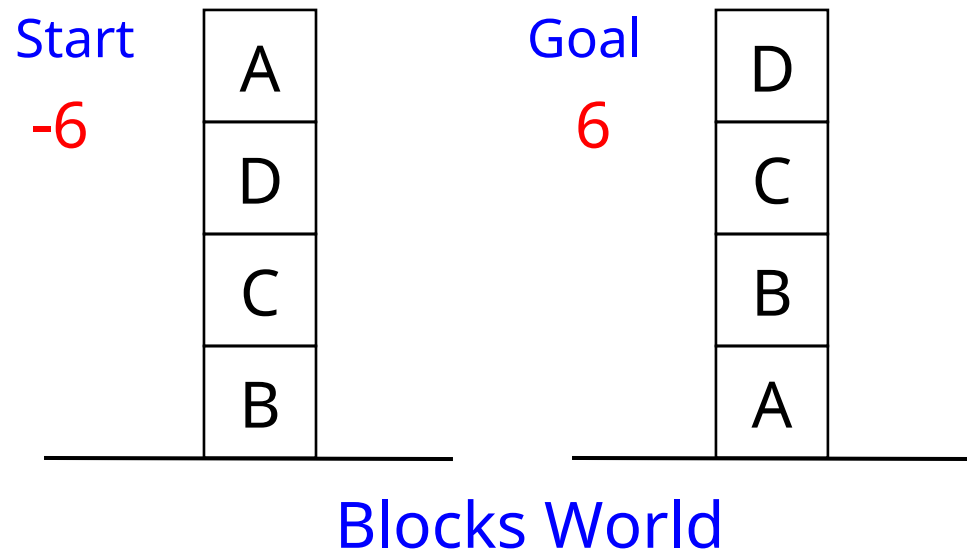
# Hill Climbing: Disadvantages



# Hill Climbing: Disadvantages



# Hill Climbing: Disadvantages

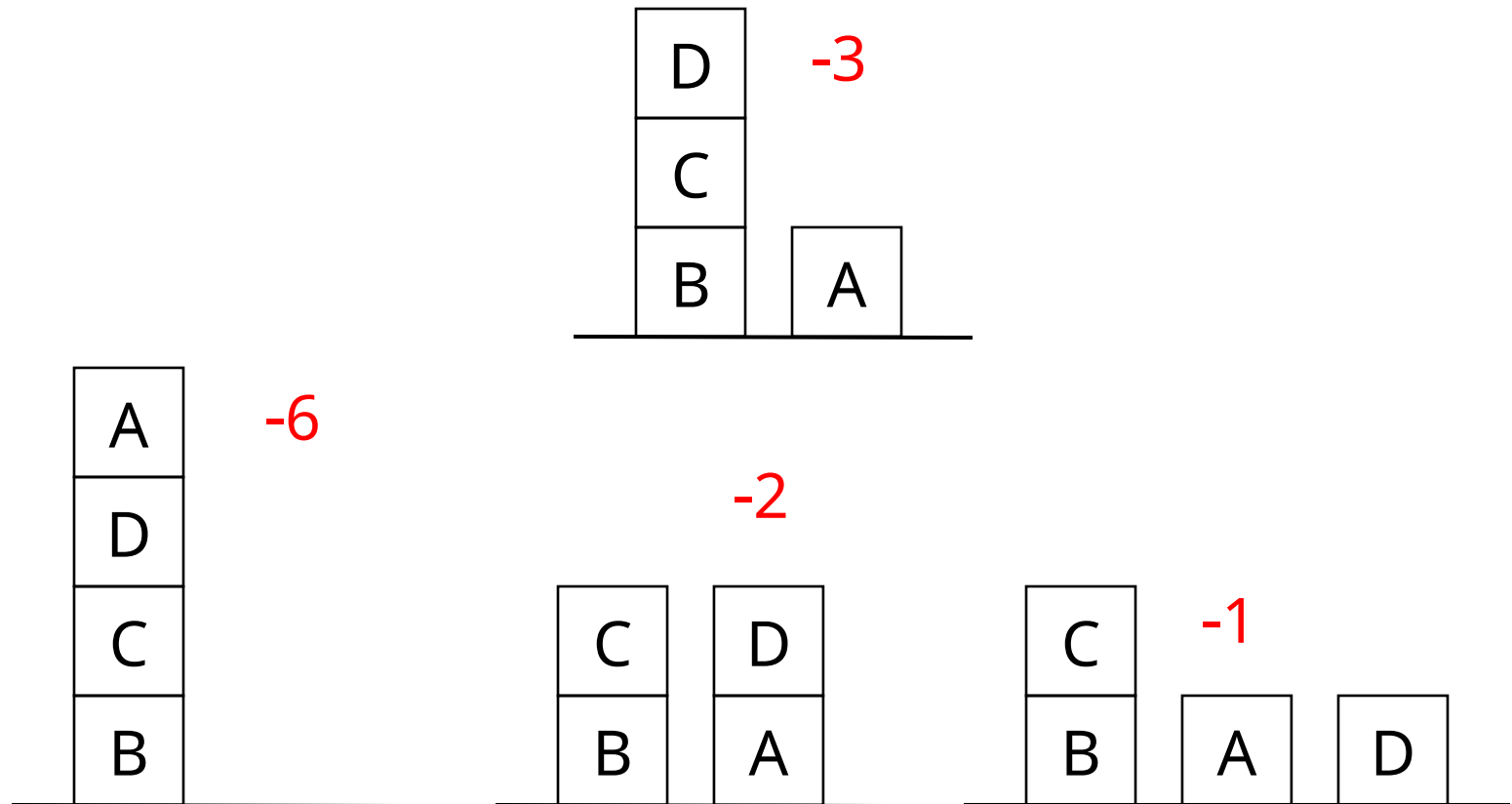


Global heuristic:

For each block that has the correct support structure: **+1** to every block in the support structure.

For each block that has a wrong support structure: **-1** to every block in the support structure.

# Hill Climbing: Disadvantages



# Hill Climbing: Conclusion

- Can be **very inefficient** in a large, rough problem space.
- Global heuristic may have to pay for **computational complexity**.
- **Often useful** when combined with other methods.

# Simulated Annealing

- A variation of hill climbing in which, at the beginning of the process, some downhill moves may be made.
- To do enough exploration of the whole space early on, so that the final solution is relatively insensitive to the starting state.
- Lowering the chances of getting caught at a local maximum, or plateau, or a ridge.

# Simulated Annealing

## Physical Annealing

- Physical substances are melted and then **gradually cooled** until some solid state is reached.
- The goal is to produce a **minimal-energy** state.
- **Annealing schedule**: if the temperature is lowered sufficiently slowly, then the goal will be attained.
- Nevertheless, there is some probability for a transition to a higher energy state:  $p = e^{-\Delta E/kT}$ 
  - $\Delta E$  : positive change in the energy level
  - T : temperature
  - k : Boltzmann's Constant

# Simulated Annealing

## Algorithm

1. Evaluate the initial state.
  2. Loop until a solution is found or there are no new operators left to be applied:
    - Set  $T$  according to an annealing schedule
    - Selects and applies a new operator
- Evaluate the new state:

goal  $\rightarrow$  quit

$\Delta E = \text{Val}(\text{current state}) - \text{Val}(\text{new state})$

$\Delta E < 0 \rightarrow$  new current state

else  $\rightarrow$  new current state with probability  $e^{-\Delta E/KT}$ .

# Difference from Simple Hill Climbing

1. Annealing schedule must be maintained
2. Moves to worse states may be accepted
  - It is a good idea to maintain, current state, the best state found so far.
  - Then, if the final state is worse than that earlier state, the earlier state is still available

# Best-First Search

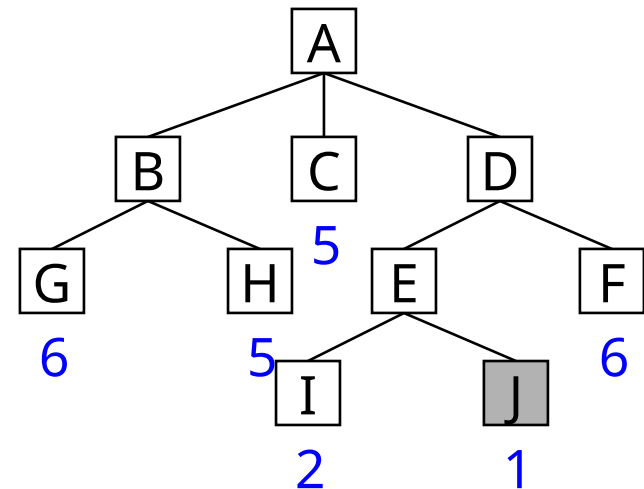
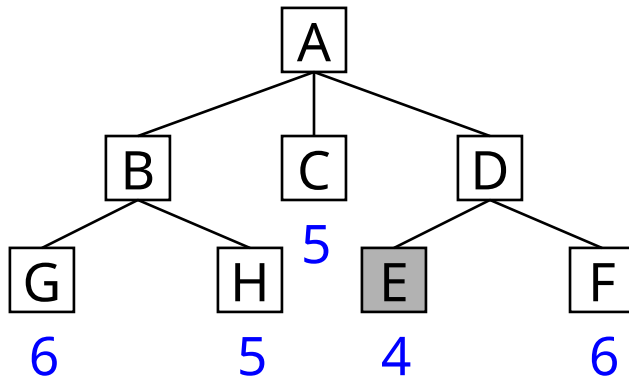
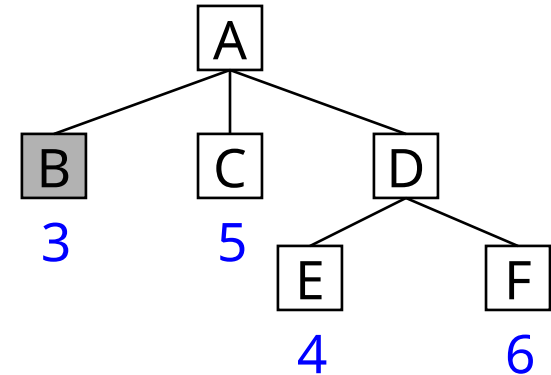
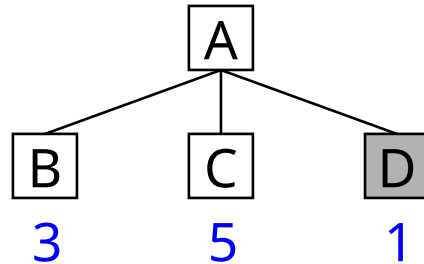
- A way of Combining the advantage of both DFS and BFS into a single method.

# Best-First Search: OR Graphs

- **Depth-first search**: not all competing branches having to be expanded.
- **Breadth-first search**: not getting trapped on dead-end paths.
  - ⇒ Combining the two is to **follow a single path at a time**, but **switch paths** whenever some competing path look more promising than the current one.

# Best-First Search: OR Graphs

A



A graph of this sort are called **OR graph**, since each of its branches represent an alternative problem-solving path.

# Best-First Search: OR Graphs

- **OPEN**: nodes that have been generated, but have not examined.

This is organized as a **priority queue**.

- **CLOSED**: nodes that have already been examined.

Whenever a new node is generated, **check** whether it has been **generated before**.

# Best-First Search

## Algorithm

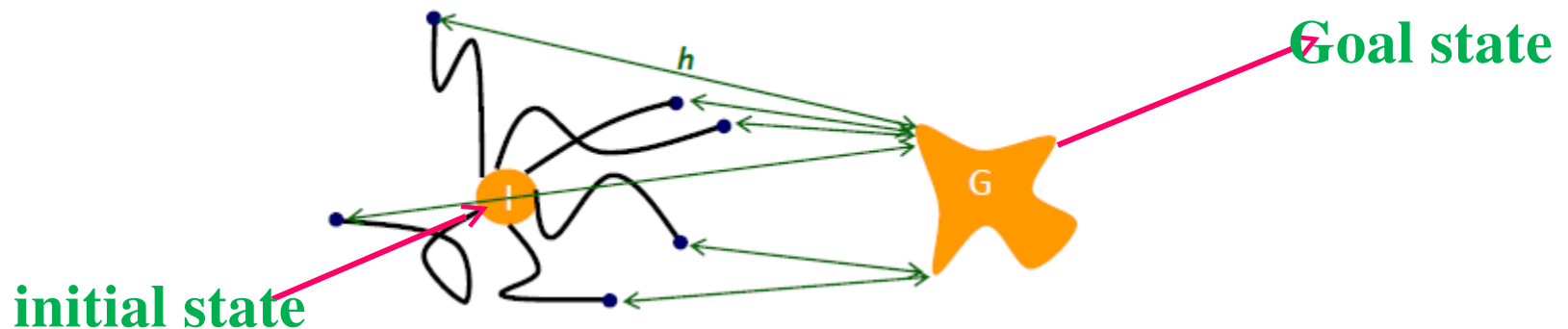
1. OPEN = {initial state}.
  2. Loop until a goal is found or there are no nodes left in OPEN:
    - Pick the best node in OPEN
    - Generate its successors
- For each successor:
- new → evaluate it, add it to OPEN, record its parent
  - generated before → change parent, update successors

# Best-First Search

- Some authors have used "best-first search" to refer specifically to a search with a heuristic that attempts to predict **how close the end of a path is to a solution**, so that paths which are judged to be **closer to a solution are extended first**.
- This specific type of search is called **greedy best-first search**
- Special cases:
  - greedy best-first search
  - A\* search

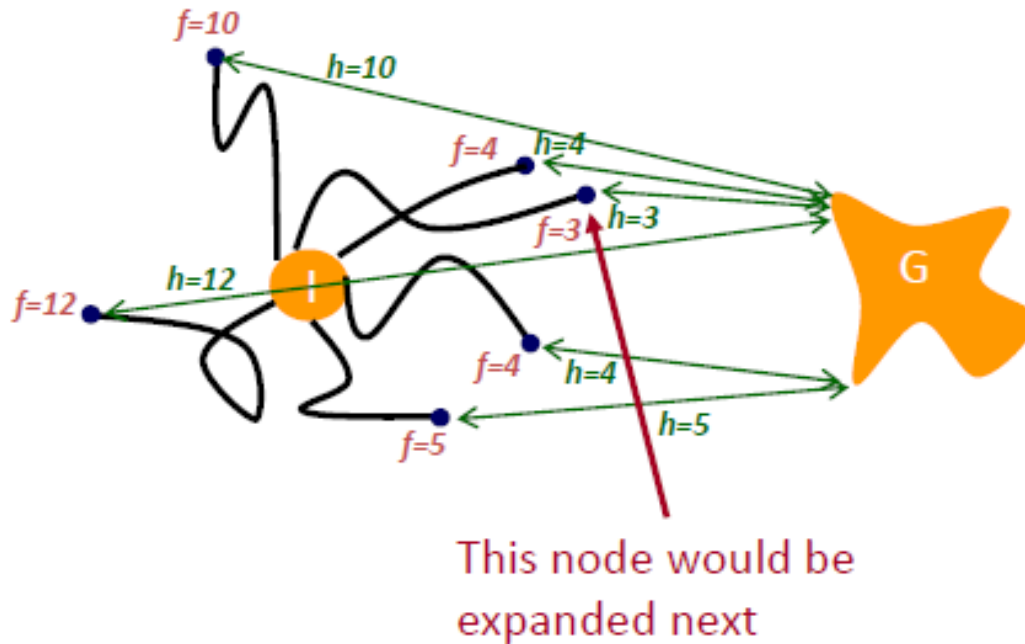
# Greedy Best-First Search

- Evaluation function  $f(n) = h(n)$  (heuristic)  
= estimate of cost from  $n$  to goal
- Greedy best-first search expands the node that **appears** to be closest to goal
- e.g., hSLD( $n$ ) = straight-line distance from  $n$  to Bucharest



# Greedy Best-First Search

- $f(n) = h(n)$  : Expand node that appears to be closest to the goal

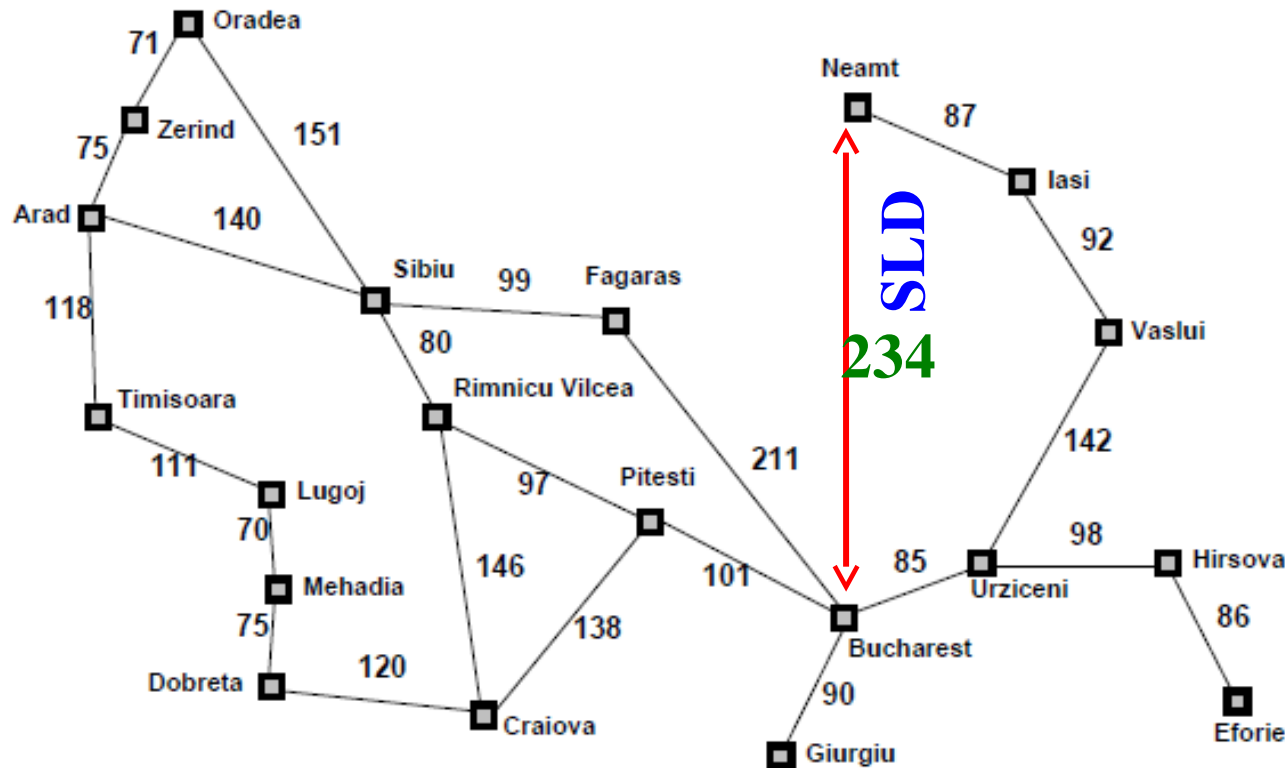


# Greedy Best-First Tree Search

## Example

Go from Arad to Bucharest

- $f(n) = h_{SLD}(n) =$  straight-line distance from  $n$  STATE to Bucharest

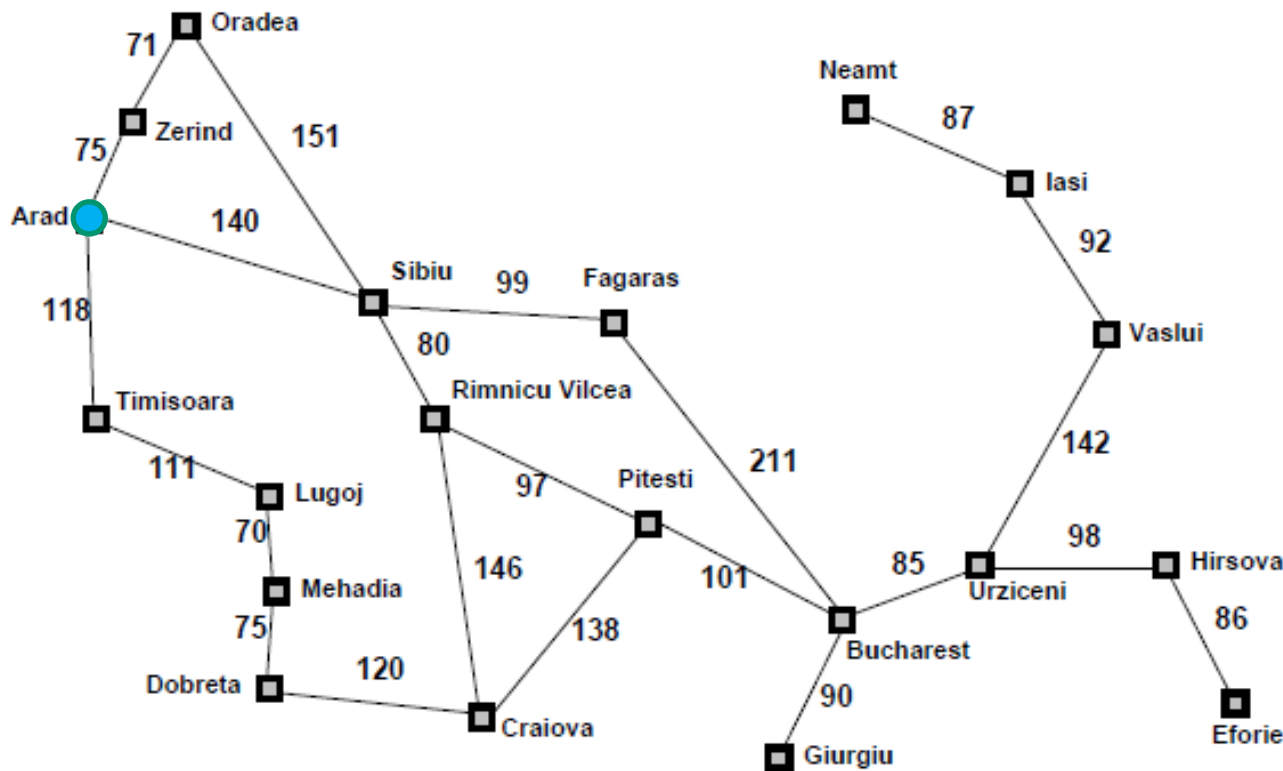
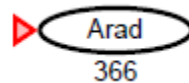


### h(n)

Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

# Greedy Best-First Tree Search Example

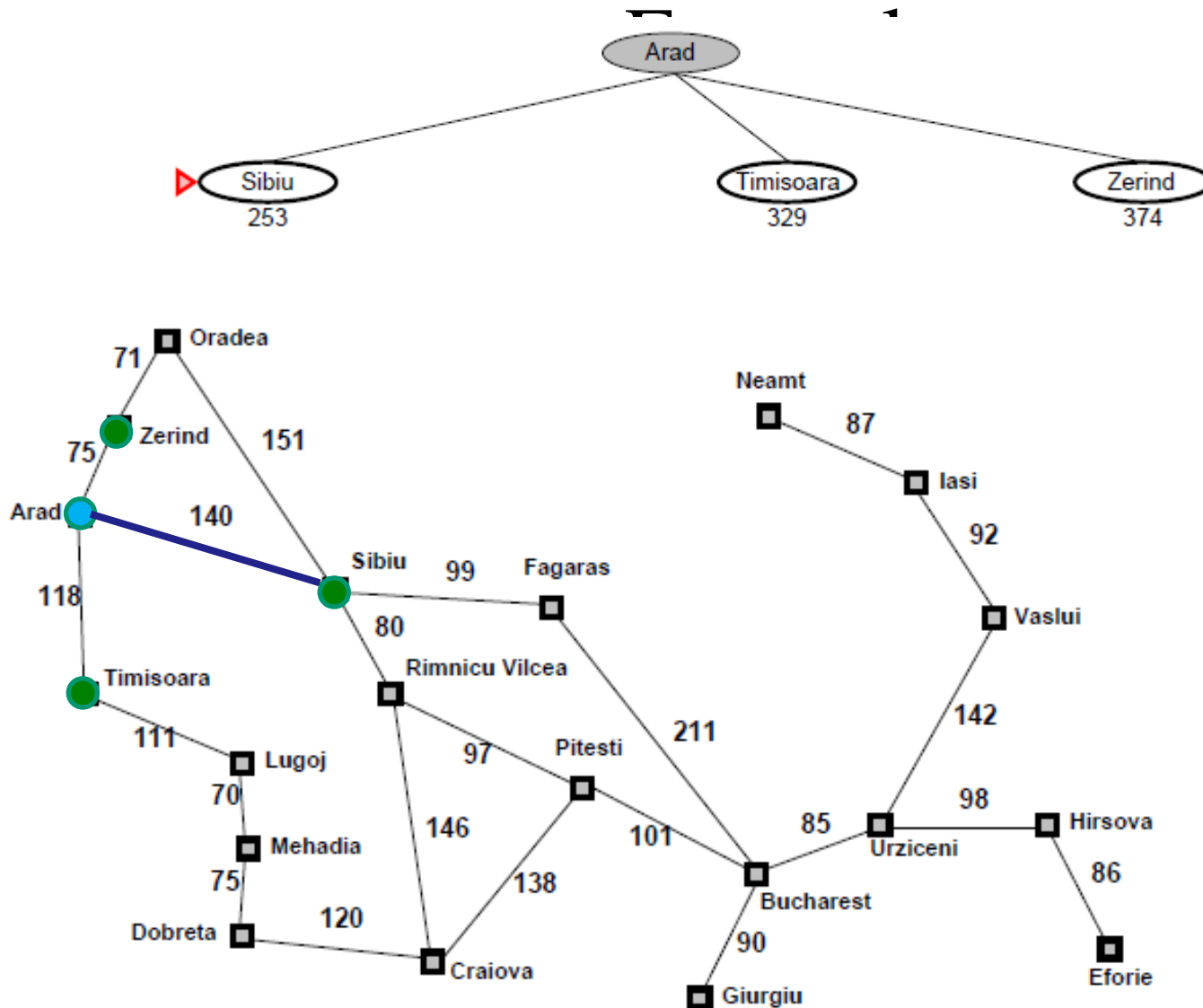


$h(n)$

Straight-line distance  
to Bucharest

Arad	366	→
Bucharest	0	
Craiova	160	
Dobreta	242	
Eforie	161	
Fagaras	178	
Giurgiu	77	
Hirsova	151	
Iasi	226	
Lugoj	244	
Mehadia	241	
Neamt	234	
Oradea	380	
Pitesti	98	
Rimnicu Vilcea	193	
Sibiu	253	
Timisoara	329	
Urziceni	80	
Vaslui	199	
Zerind	374	

# Greedy Best-First Tree Search

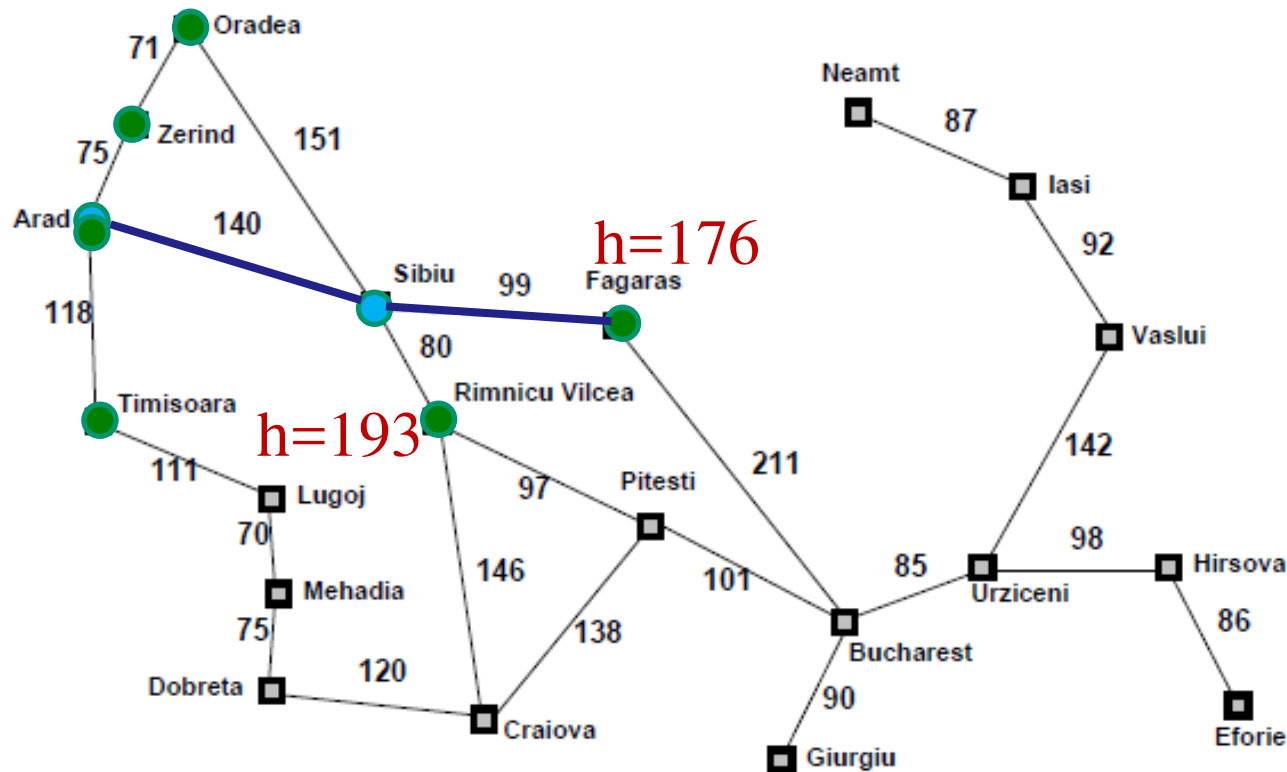
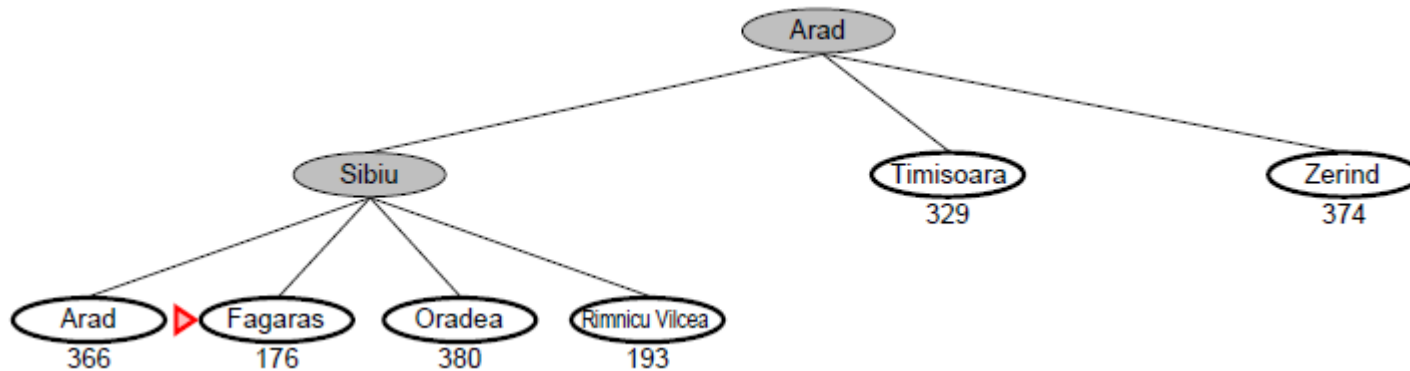


$h(n)$

Straight-line distance to Bucharest

Arad	366	←
Bucharest	0	
Craiova	160	
Dobreta	242	
Eforie	161	
Fagaras	178	
Giurgiu	77	
Hirsova	151	
Iasi	226	
Lugoj	244	
Mehadia	241	
Neamt	234	
Oradea	380	
Pitesti	98	
Rimnicu Vilcea	193	
Sibiu	253	←
Timisoara	329	←
Urziceni	80	
Vaslui	199	
Zerind	374	←

# Greedy Best-First Tree Search Example

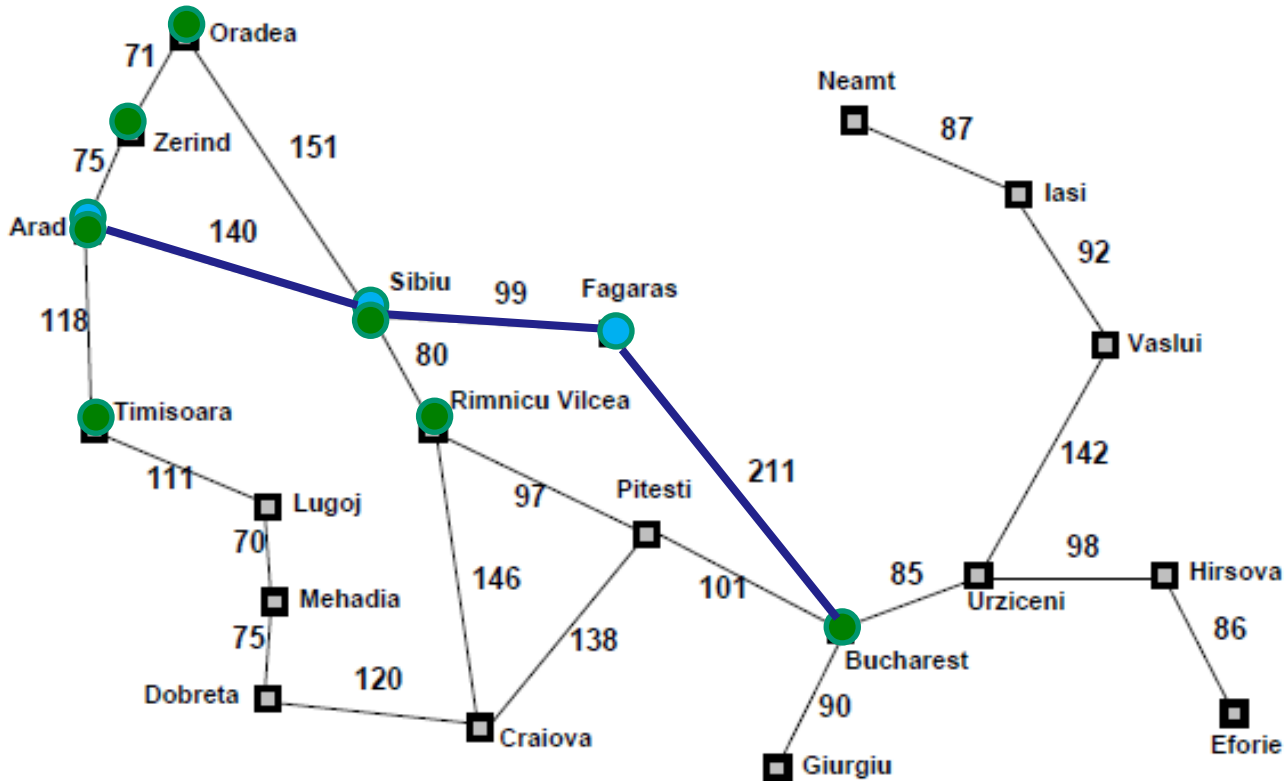
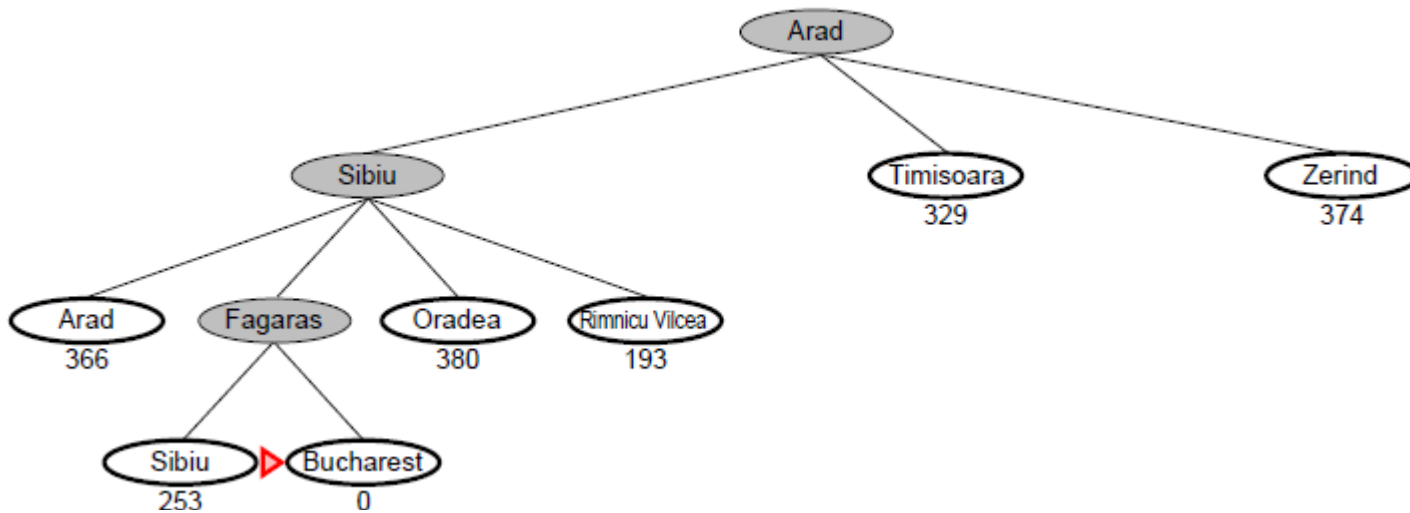


**$h(n)$**

Straight-line distance to Bucharest

Arad	366	←
Bucharest	0	
Craiova	160	
Dobreta	242	
Eforie	161	
Fagaras	178	←
Giurgiu	77	
Hirsova	151	
Iasi	226	
Lugoj	244	
Mehadia	241	
Neamt	234	
Oradea	380	←
Pitesti	98	
Rimnicu Vilcea	193	←
Sibiu	253	←
Timisoara	329	←
Urziceni	80	
Vaslui	199	
Zerind	374	←

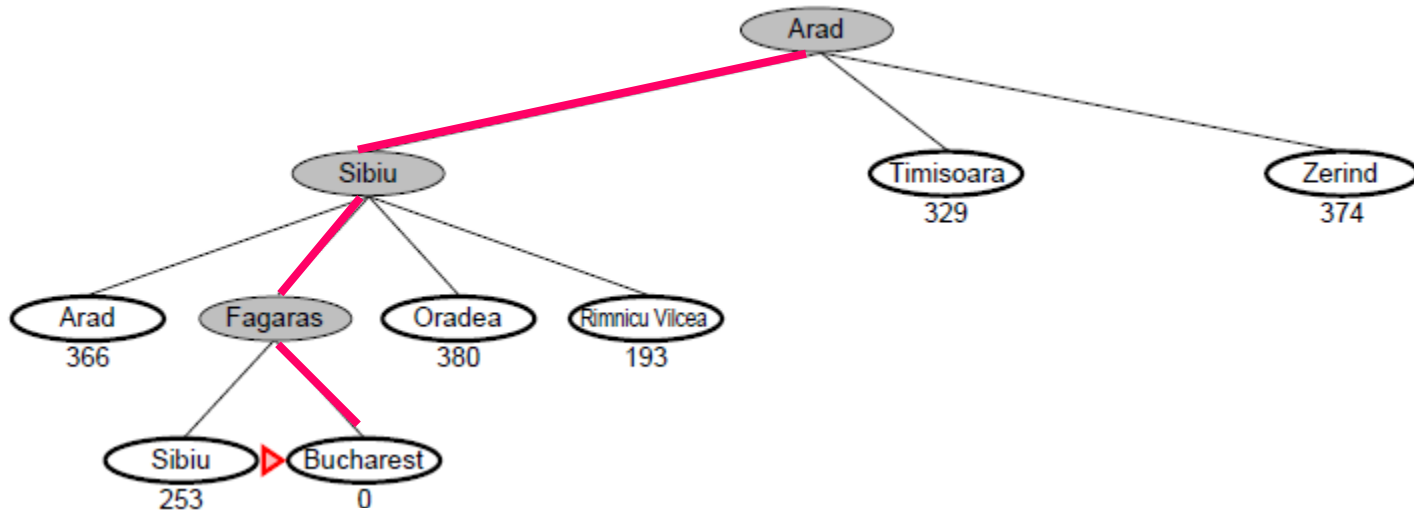
# Example



## $h(n)$

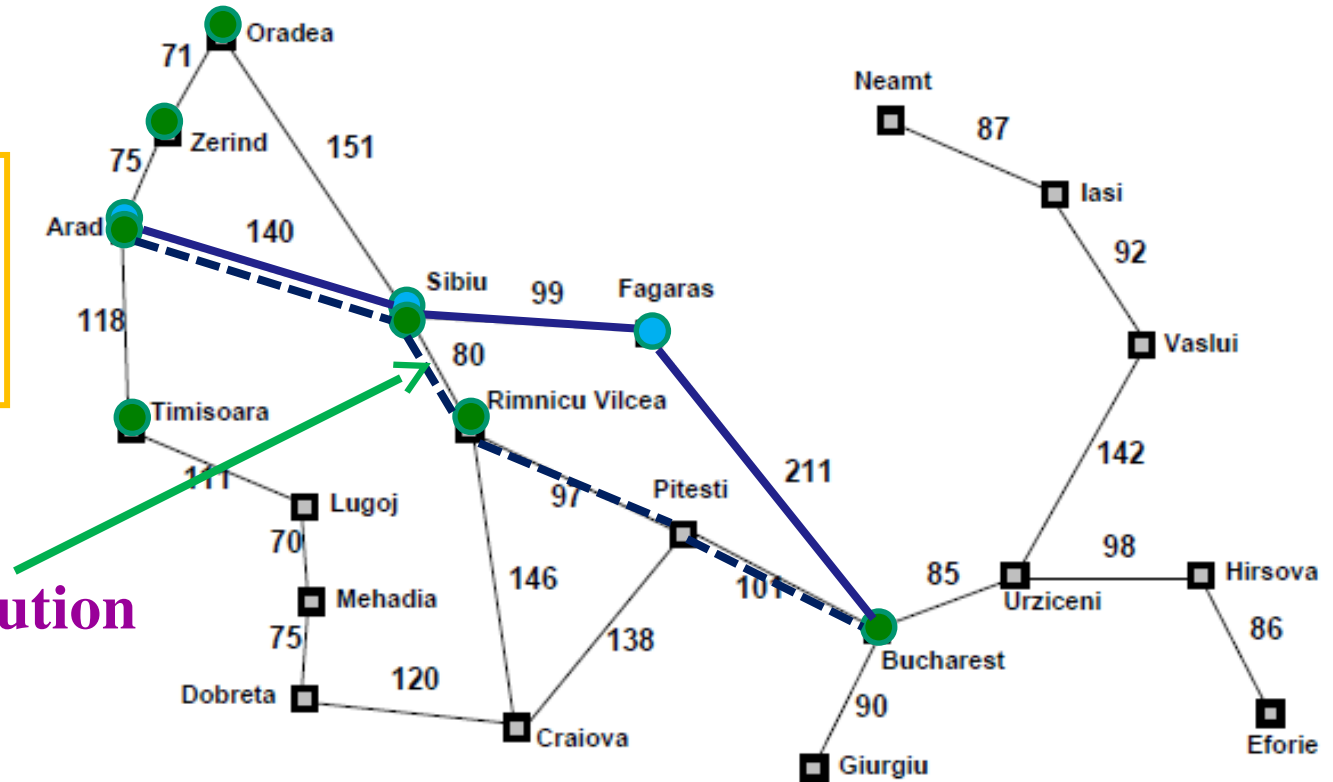
Straight-line distance to Bucharest

Arad	366	←
Bucharest	0	←
Craiova	160	
Dobreta	242	
Eforie	161	
Fagaras	178	←
Giurgiu	77	
Hirsova	151	
Iasi	226	
Lugoj	244	
Mehadia	241	
Neamt	234	
Oradea	380	←
Pitesti	98	
Rimnicu Vilcea	193	←
Sibiu	253	←
Timisoara	329	←
Urziceni	80	
Vaslui	199	
Zerind	374	←



**Solution  
turns out not  
to be optimal**

**Optimal solution**



# Properties of greedy best-first search

- Complete? No – can get stuck in loops, e.g., Iasi → Neamt → Iasi → Neamt →
- Time?  $O(b^m)$ , but a good heuristic can give dramatic improvement
- Space?  $O(b^m)$  -- keeps all nodes in memory
- Optimal? No
- Complete in finite space with repeated-state checking

# Greedy Best-First Search

- Greedy search:  
 $h(n)$  = estimated cost of the cheapest path from node  $n$  to a goal state.

Neither optimal nor complete

- Uniform-cost search:  
 $g(n)$  = cost of the cheapest path from the initial state to node  $n$ .

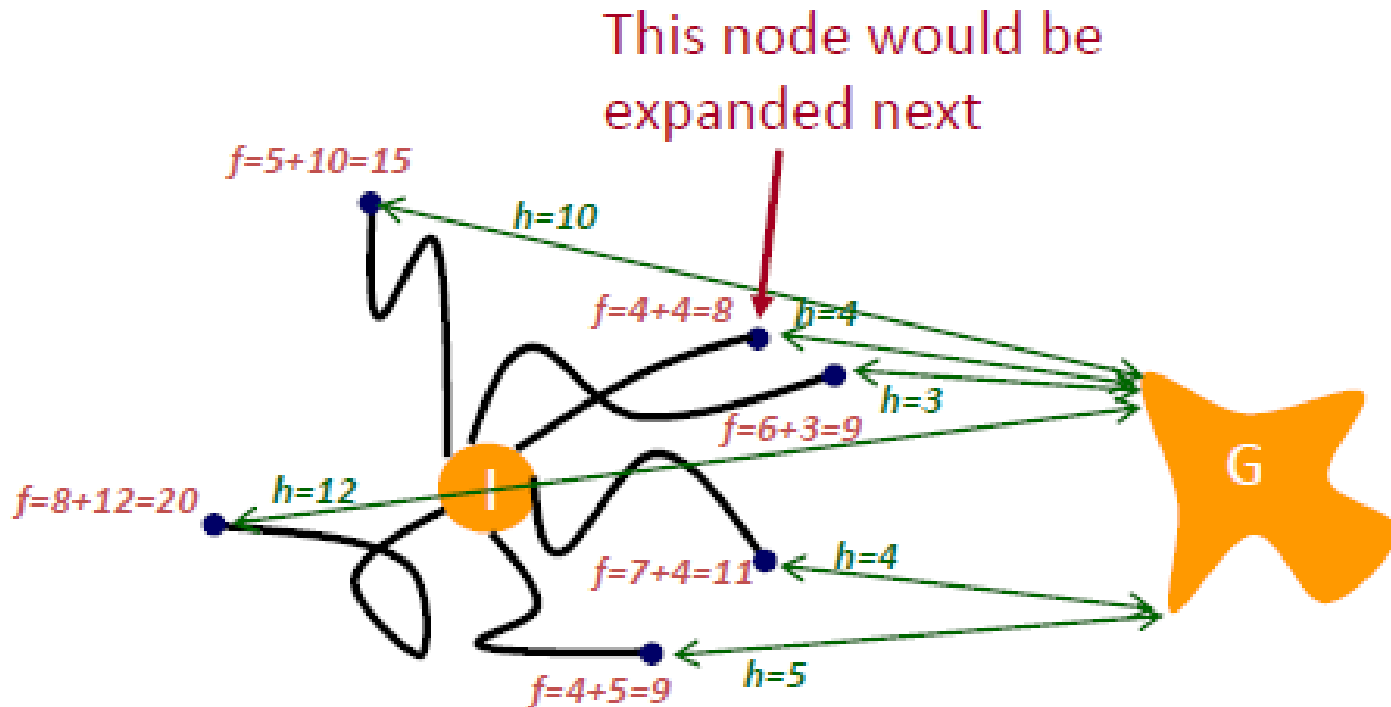
Optimal and complete, but very inefficient

# A\* search

- **Idea:** avoid expanding paths that are already expensive
  - include cost of reaching node
  - Evaluation function  $f(n) = g(n) + h(n)$ 
    - $g(n)$  = *cost of reaching  $n$*
    - $h(n)$  = *estimated cost of reaching goal from state of  $n$*
    - $f(n)$  = *estimated cost of the cheapest path to a goal state that goes through path of  $n$*

# A\* Search

- $f(n) = g(n) + h(n)$  : Expand node that appears to be on cheapest paths to the

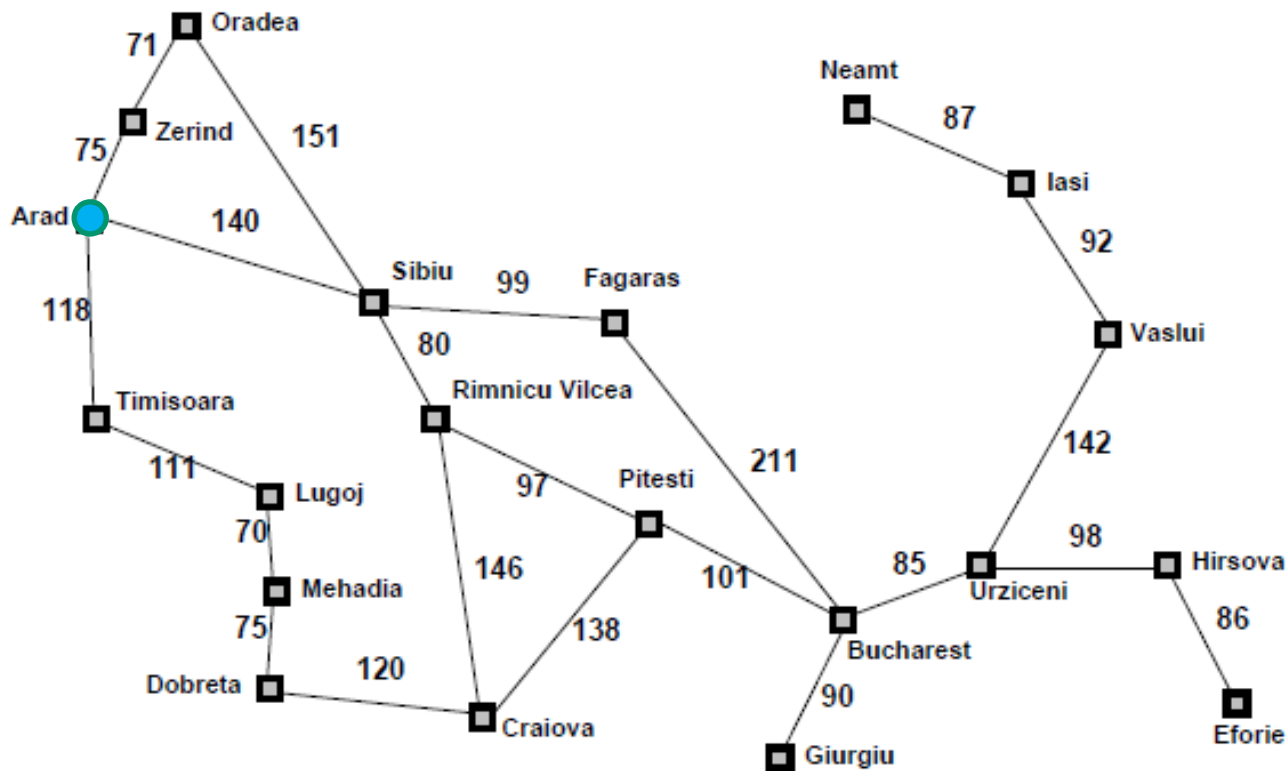
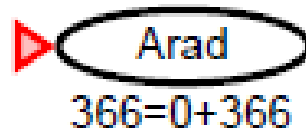


# A\* search example

Go from Arad to Bucharest

- $f(n) = g(n) + h_{SLD}(n)$

# A\* search example

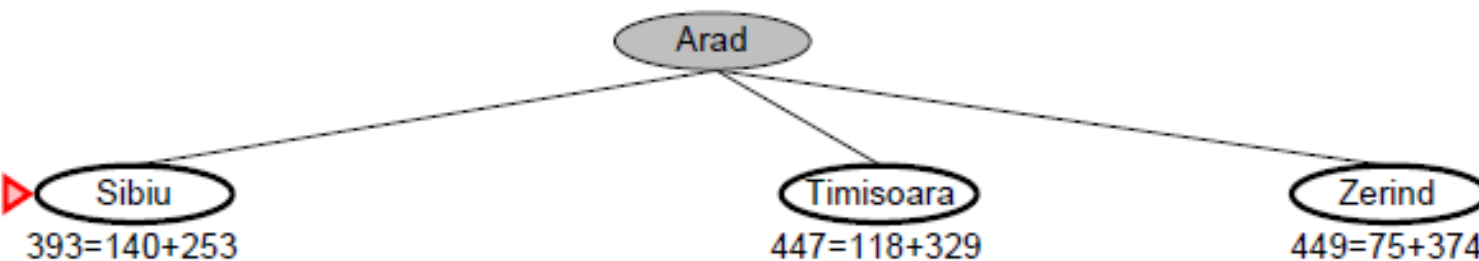


## h(n)

Straight-line distance  
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

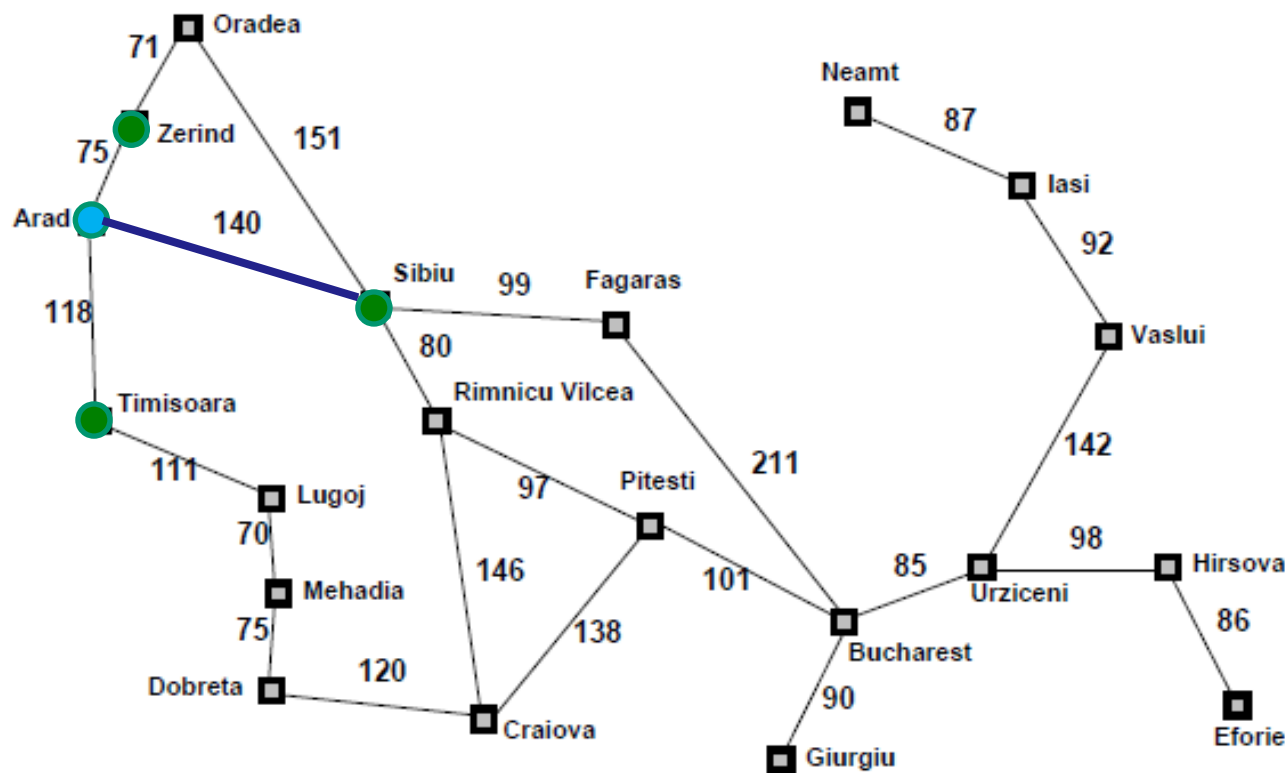
# A\* search example



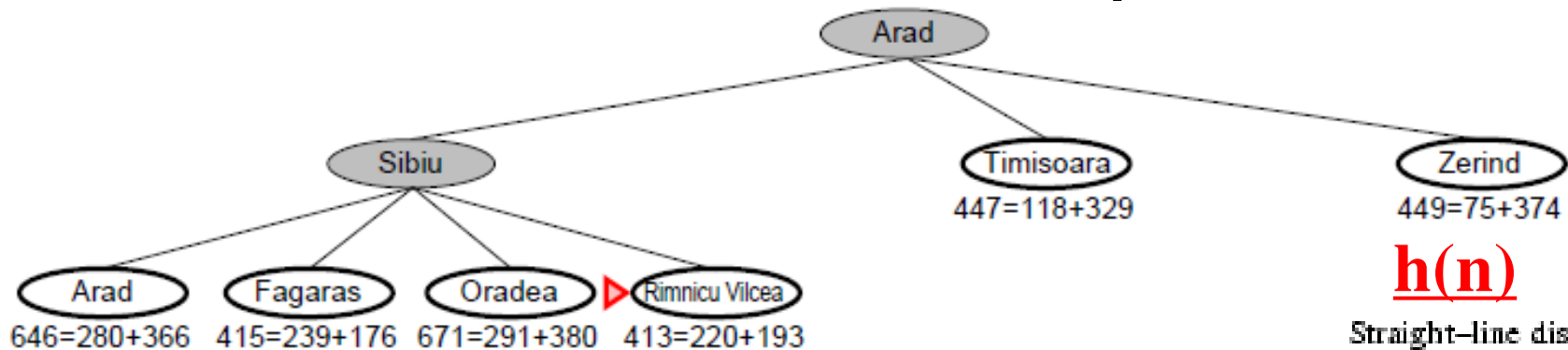
**h(n)**

Straight-line distance  
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



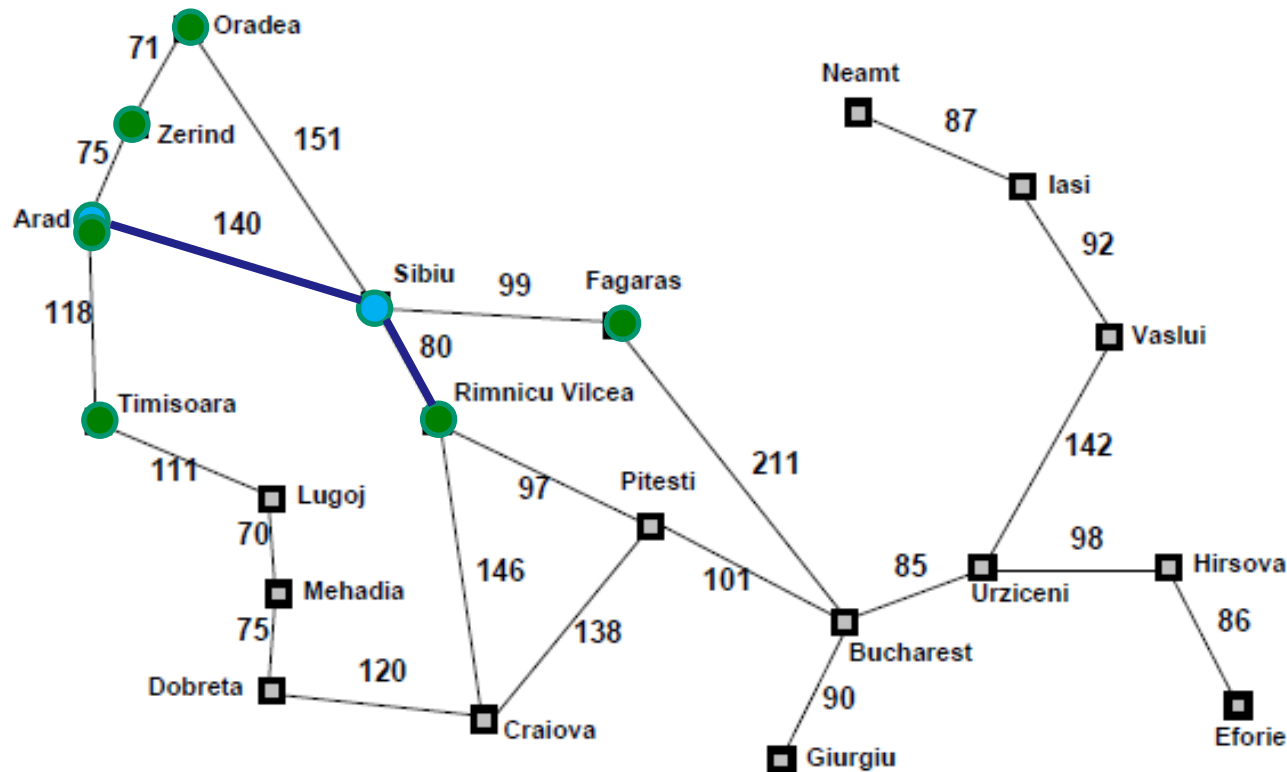
# A\* search example

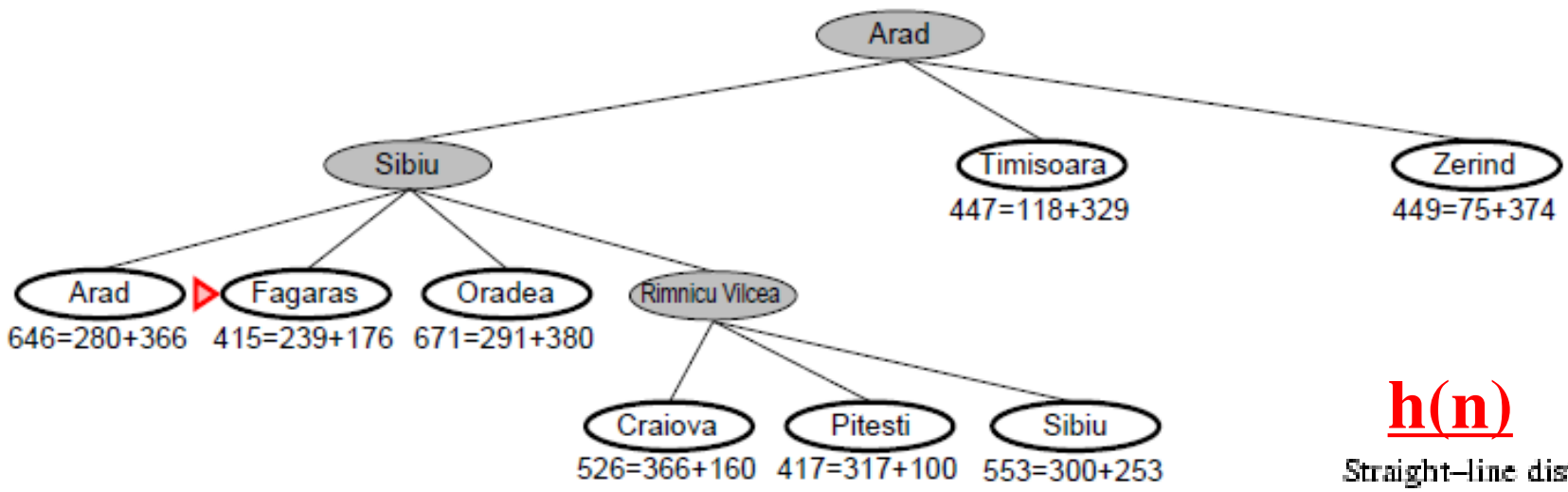


**h(n)**

Straight-line distance  
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

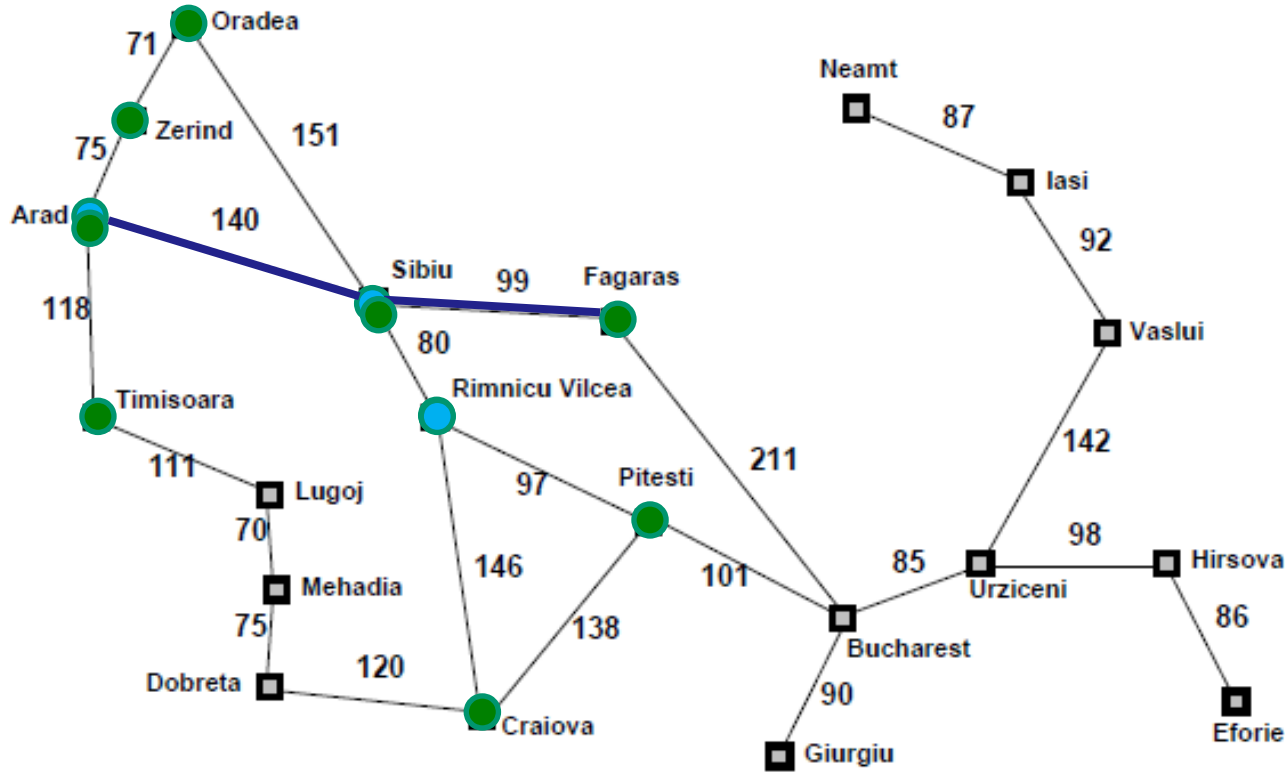


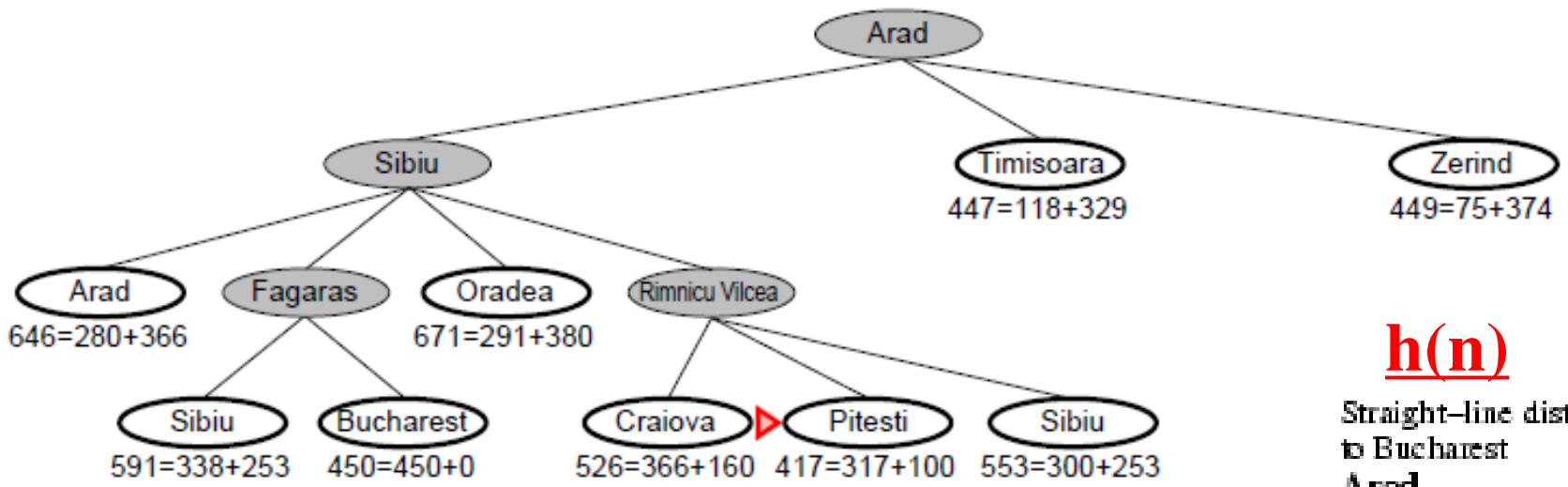


## $h(n)$

Straight-line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

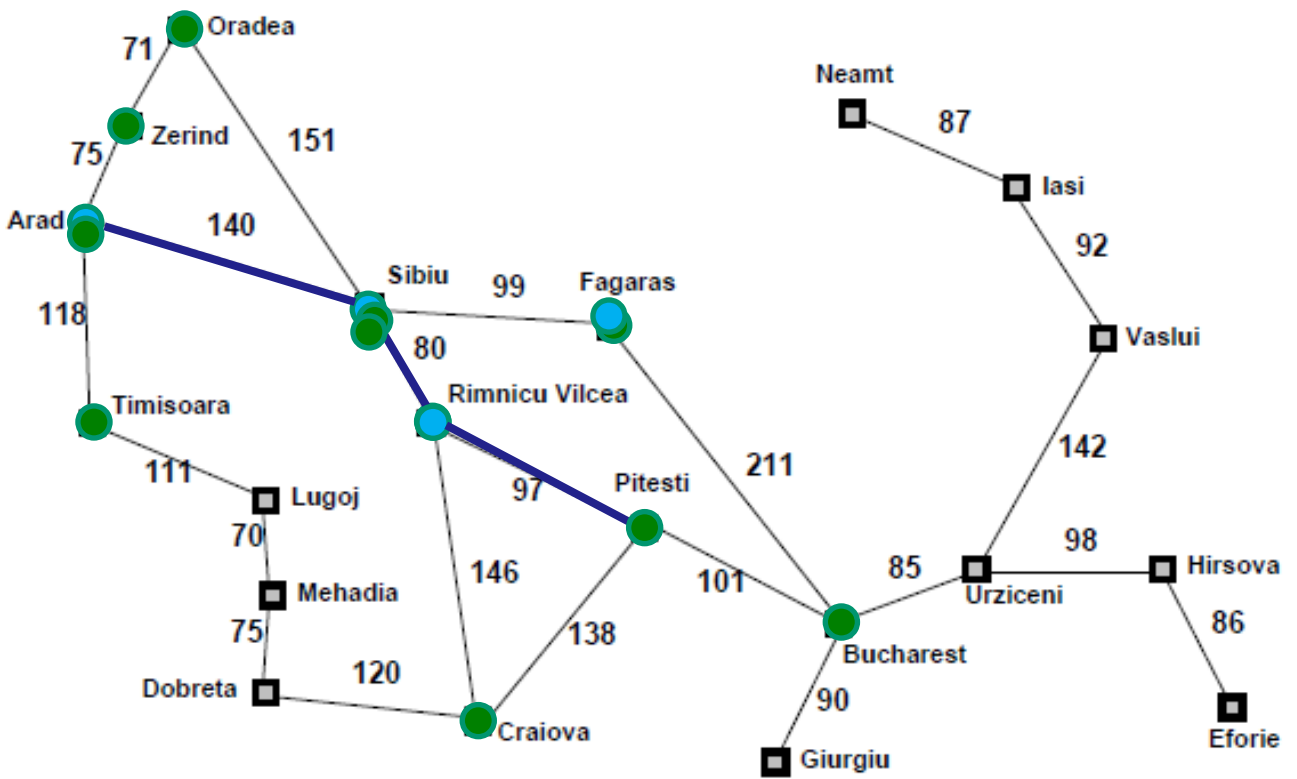


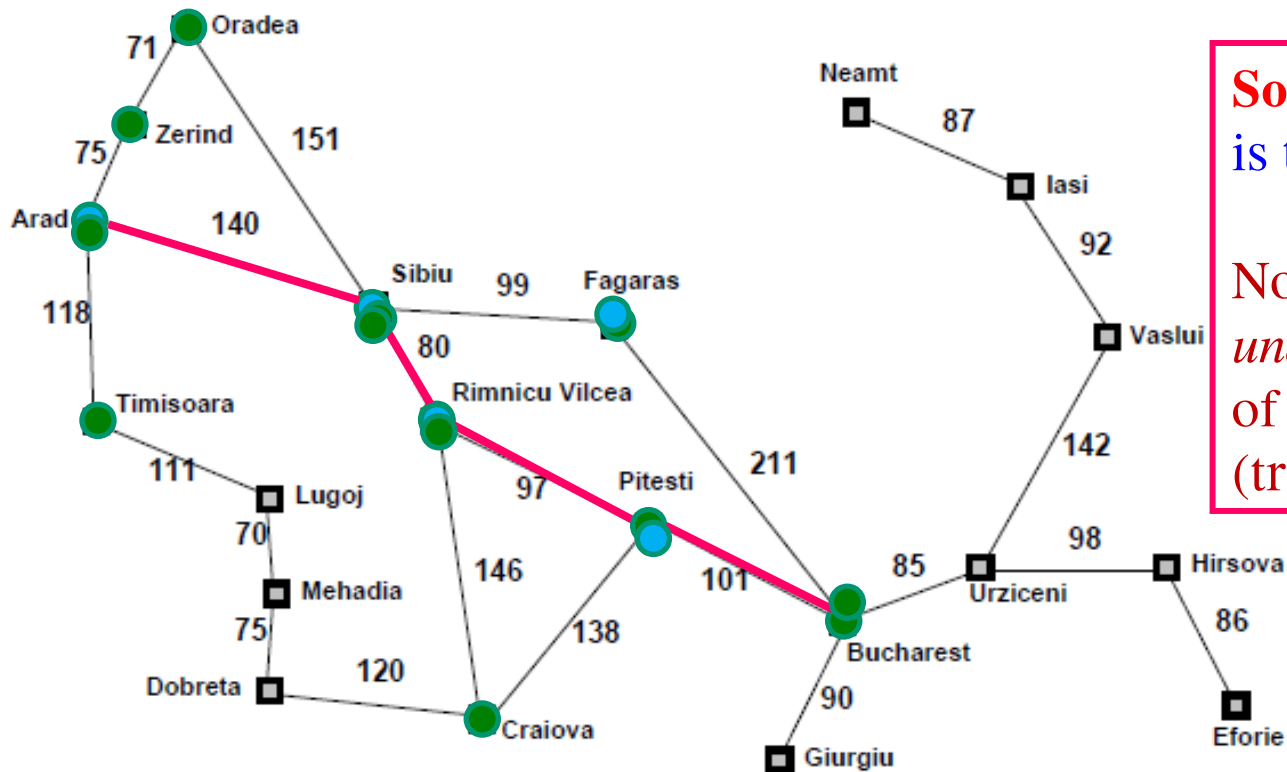
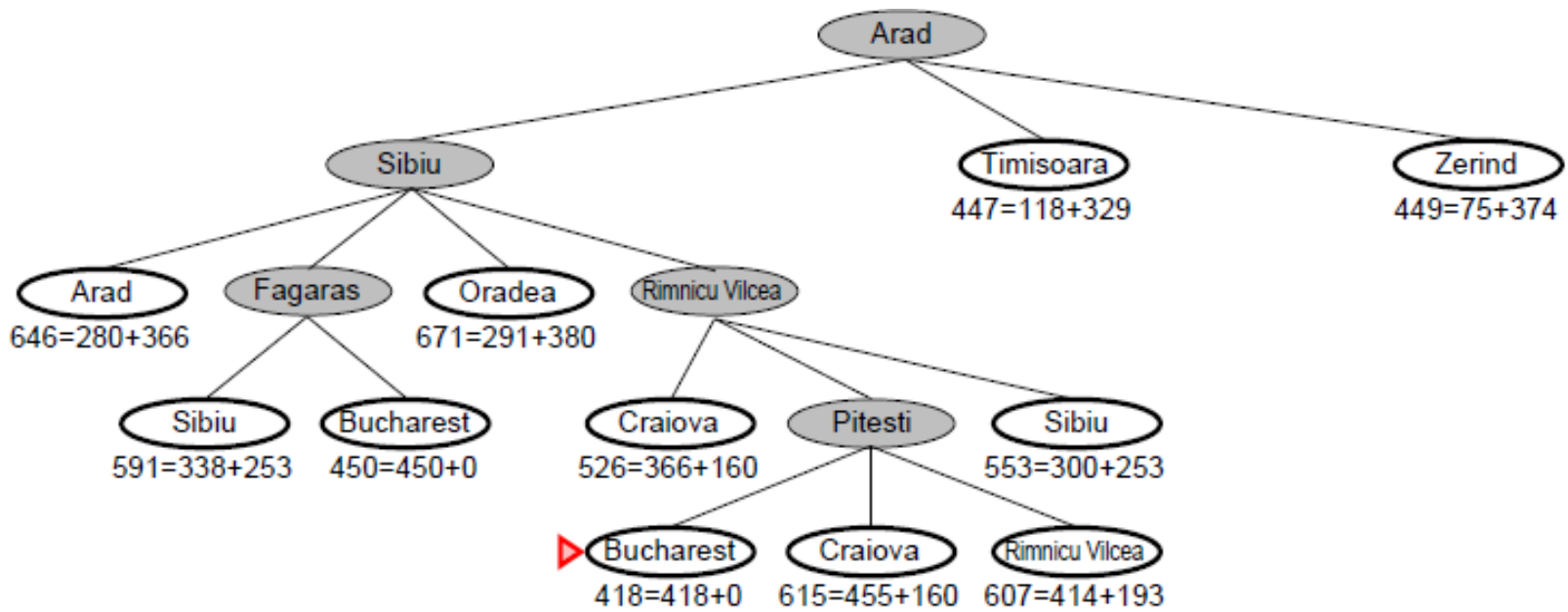


## h(n)

Straight-line distance to Bucharest

<b>Arad</b>	366
<b>Bucharest</b>	0
<b>Craiova</b>	160
<b>Dobreta</b>	242
<b>Eforie</b>	161
<b>Fagaras</b>	176
<b>Giurgiu</b>	77
<b>Hirsova</b>	151
<b>Iasi</b>	226
<b>Lugoj</b>	244
<b>Mehadia</b>	241
<b>Neamt</b>	234
<b>Oradea</b>	380
<b>Pitesti</b>	10
<b>Rimnicu Vilcea</b>	193
<b>Sibiu</b>	253
<b>Timisoara</b>	329
<b>Urziceni</b>	80
<b>Vaslui</b>	199
<b>Zerind</b>	374





**Solution is optimal!**  
is this a coincidence?

No, when  $h(n)$  underestimates the cost of reaching a goal, A\* (tree search) is optimal

# Admissible heuristic

- Let  $h^*(N)$  be the **true** cost of the optimal path from  $N$  to a goal node

- Heuristic  $h(N)$  is **admissible** if:

$$0 \leq h(N) \leq h^*(N)$$

- An admissible heuristic is always **optimistic**

# Admissible heuristics

- For the 8-puzzle: If we want to find the shortest solutions, we need a heuristic function that never overestimates the number of steps to the goal. Here are two candidates:
- $h_1(n)$  = number of misplaced tiles
- $h_2(n)$  = total Manhattan distance (no. of squares from desired location of each tile)

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

Goal State

$$h_1(S) = ??$$

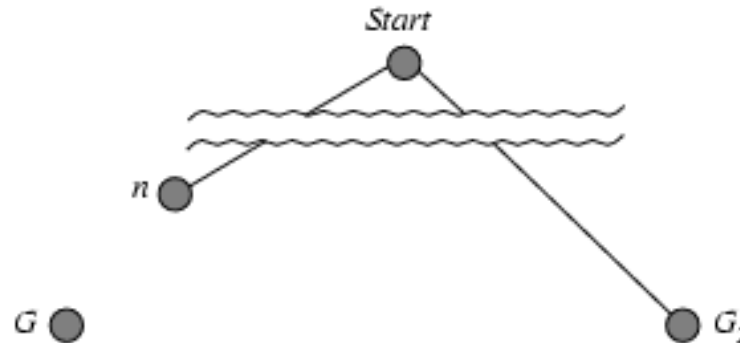
$$h_2(S) = ??$$

$$h_1(S) = ?? \quad 6$$

$$h_2(S) = ?? \quad 4+0+3+3+1+0+2+1 = 14$$

# Optimality of $A^*$ (proof)

- Suppose some suboptimal goal  $G_2$  has been generated and is in the fringe. Let  $n$  be an unexpanded node in the fringe such that  $n$  is on a shortest path to an optimal goal  $G$ .



$$f(G_2) = g(G_2)$$

$$g(G_2) > g(G)$$

$$f(G) = g(G)$$

$$f(G_2) > f(G)$$

$$\text{since } h(G_2) = 0$$

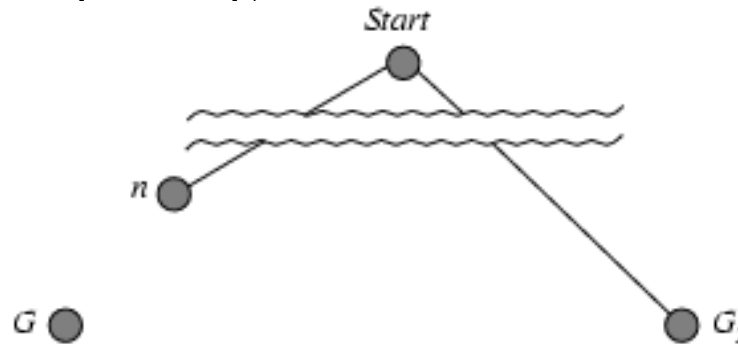
since  $G_2$  is suboptimal

$$\text{since } h(G) = 0$$

from above

# Optimality of A\* (proof)

- Suppose some suboptimal goal  $G_2$  has been generated and is in the fringe. Let  $n$  be an unexpanded node in the fringe such that  $n$  is on a shortest path to an optimal goal  $G$ .



$f(G_2)$	$> f(G)$	from above
$h(n)$	$\leq h^*(n)$	since $h$ is admissible
$g(n) + h(n)$	$\leq g(n) + h^*(n)$	
$f(n)$	$\leq f(G)$	

Hence  $f(G_2) > f(n)$ , and A\* will never select  $G_2$  for expansion

# Properties of A

- **Complete??** Yes, unless there are infinitely many nodes with  $f \leq f(G)$
- **Time??** Exponential in [relative error in  $h$  x length of soln.]
- **Space??** Keeps all nodes in memory
- **Optimal??** Yes-cannot expand  $f_{i+1}$  until  $f_i$  is finished
  - A expands all nodes with  $f(n) < C^*$
  - A expands some nodes with  $f(n) = C^*$
  - A expands no nodes with  $f(n) > C^*$

# Proof of lemma: Consistency

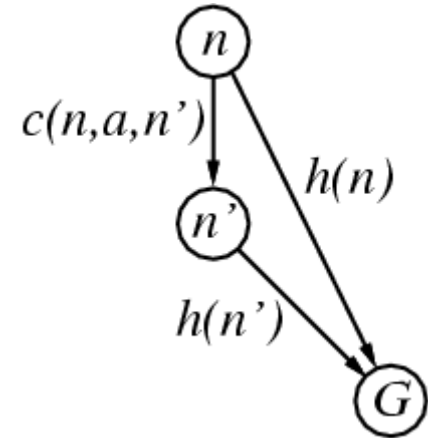
- A heuristic is consistent if

$$h(n) \leq c(n,a,n') + h(n')$$

- If  $h$  is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\geq g(n) + h(n) \\ &\geq f(n) \end{aligned}$$

i.e.,  $f(n)$  is always greater at a successor node, so once a node has been visited, it cannot be visited again at a smaller cost and thus will never be reopened



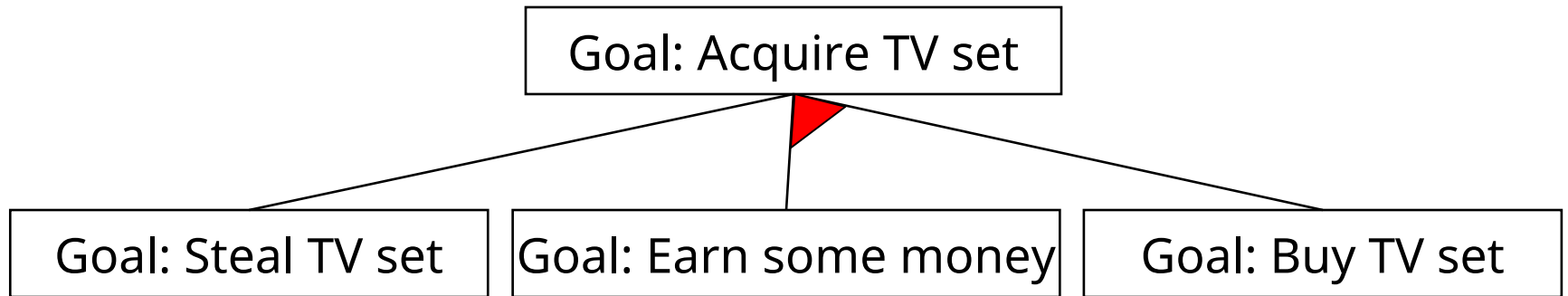
Theorem: If  $h(n)$  is consistent, A\* using GRAPH-SEARCH is optimal



# Relaxed Problems

- A problem with fewer restrictions on the actions is called a **relaxed problem**
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- Admissible heuristics can be derived from the exact solution cost of a relaxed version of the problem
- **Example:**
  - If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then  $h_1(n)$  gives the shortest solution
  - If the rules are relaxed so that a tile can move to any adjacent square, then  $h_2(n)$  gives the shortest solution
- **Key point:** the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

# Problem Reduction

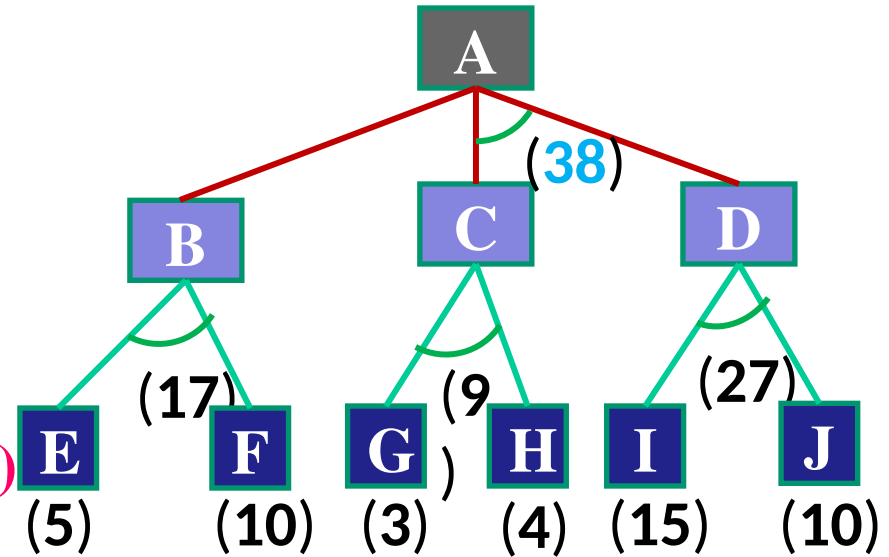


AND-OR Graphs

Algorithm AO\* (Martelli & Montanari 1973, Nilsson 1980)

# AND-OR Graph

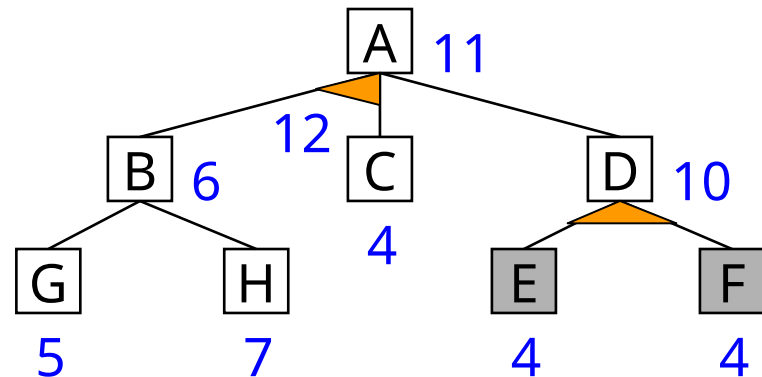
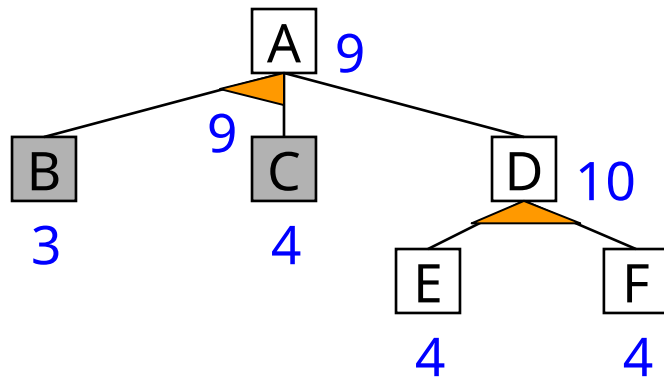
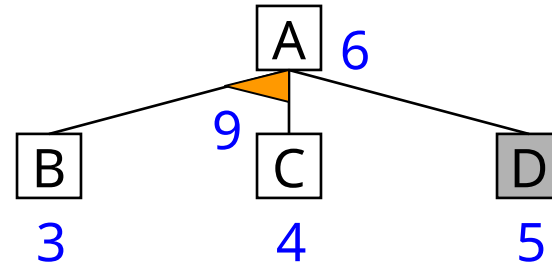
- Most promising single node: **G** ( $f^* = 3$ )
- Arc: **G-H** ( $7+2=9$ )
- But that arc is not part of the current best path since to use it, we must also use the arc: **I-J** ( $27$ )
- Path **A-B-E-F** is better ( $5+10+2+1=18$ )
- Should not expand **G** next; rather should examine either **E/F**



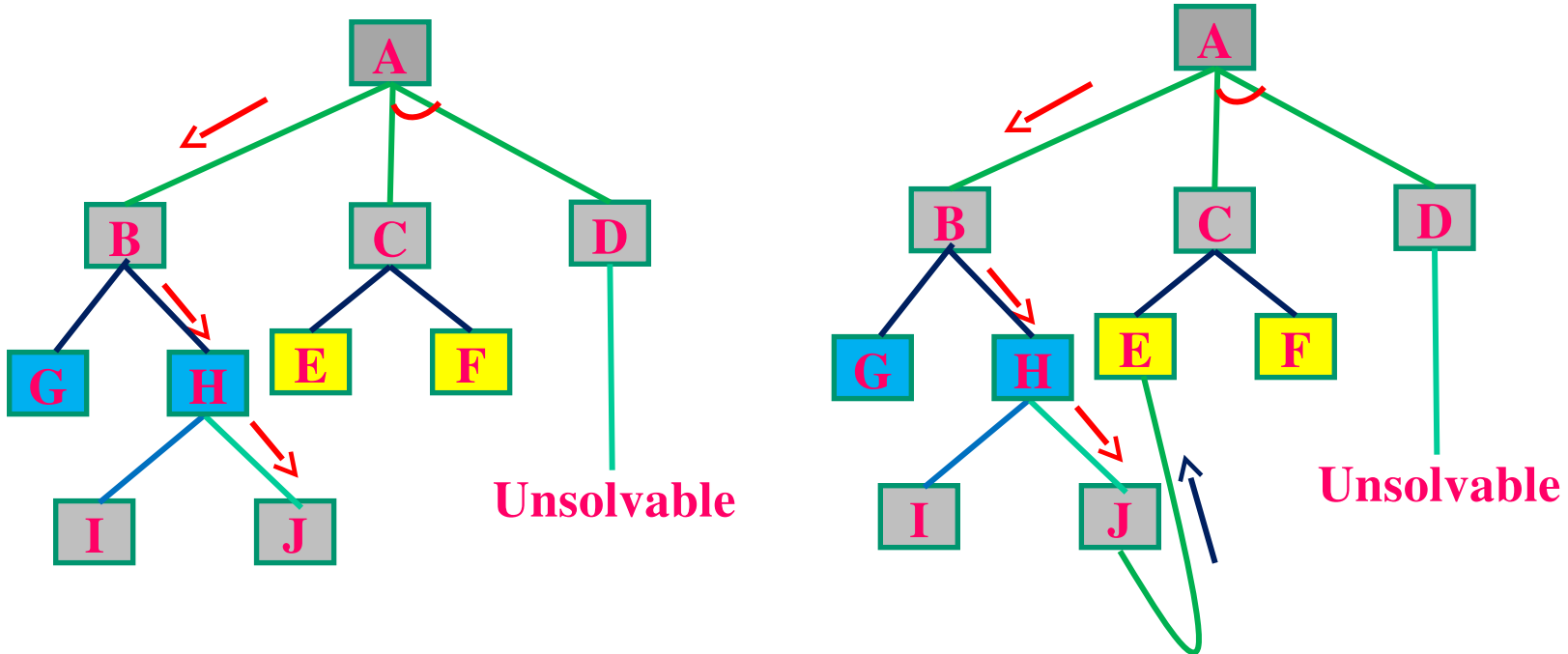
# Problem Reduction

1. Initialize the graph to the starting node
2. Loop until the starting node is labeled **SOLVED** or until its cost goes above **FUTILITY**
  - (a) Traverse the graph, starting at the initial node & following the current best path, & accumulate the set of nodes that are on that path & **have not yet been expanded/labeled as solved**
  - (b) Pick one of these unexpanded nodes & expand it. if **there are no successors**, assign **FUTILITY** as **the value of this node**. Otherwise, add its successors to the graph & for each of them compute  $f^*$  (use only  $h^*$  & ignore  $g$ ). If  $f^*$  of any node is 0, mark that node as **SOLVED**
  - (c) **Change the  $f^*$  estimate of the newly expanded node to reflect the new info provided by its successors**
    - propagate this change backward through the graph
    - if any node contains a successor are whose **descendants are all solved**, label the node itself as **SOLVED**
    - at each node that is visited while going up the graph, decide which of its **successor arcs** is the **most promising** & mark it as part of the current best path
      - This may cause the current best path to change
      - This propagation of revised cost estimates back up the tree was not necessary in the BFS algo because only unexpanded nodes were examined
      - **But now expanded nodes must be reexamined so that the best current path can be selected**
      - Thus it is important that their  $f^*$  value estimates available

# Problem Reduction(Example)



# Problem Reduction: Limitation



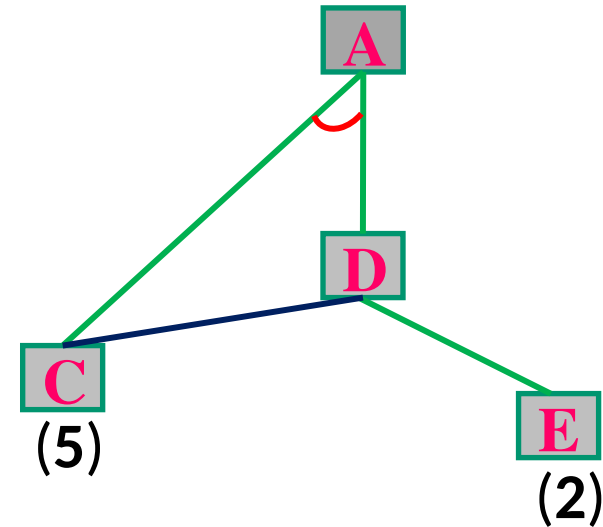
-suppose J is expanded at the next step  
-that one of its successors in node E

□ **Long path may be better**

-This new path to E is longer than the previous path to E going through C  
-but since the path through C will only lead to a solution if there is also a solution to D, which we know there is not, the path through J is better

# Problem Reduction: Limitation

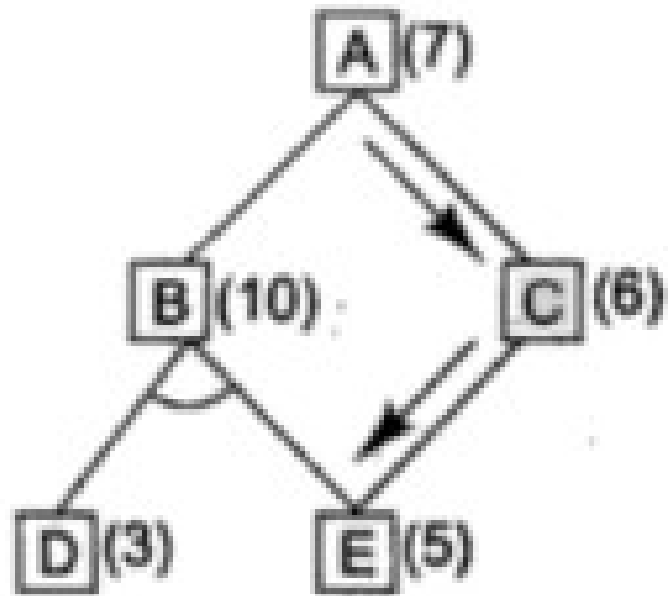
- Fails to take into account any interaction between sub goals
- Both node C/E ultimately lead to a solution, algorithm will report a complete solution that includes both of them
- AND-OR graph states that for A to be solved, both C/D must be solved
- The algo considers the solution of D as a completely separate process from the solution of C
- Looking just at the alternatives from D, E is the **best path**
- But it turns out that C is necessary anyway, so it would be better also to use it to satisfy D
- But the algo does not consider such interactions, it will find a non-optimal path.



# AO\* Algorithm

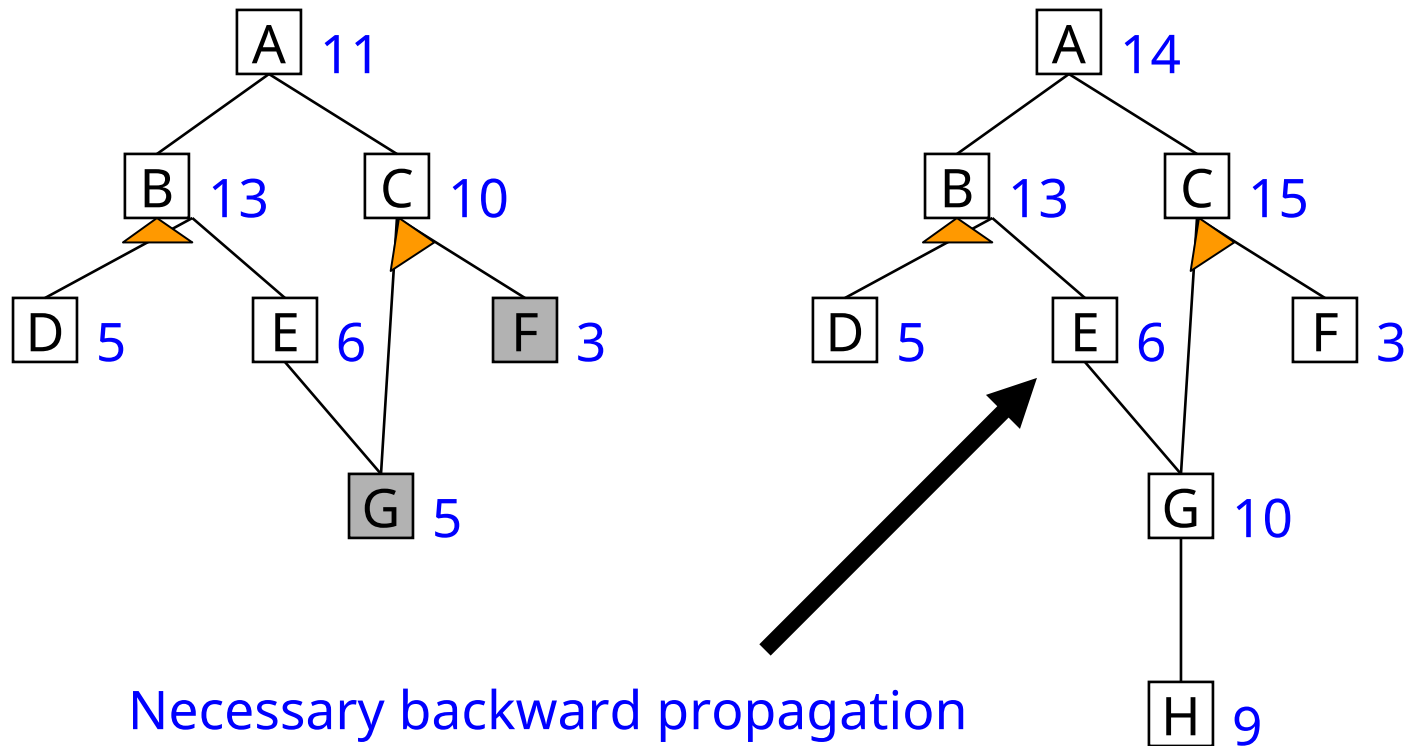
- Algorithm for searching AND-OR graph is called AO\*
- Here single structure G, representing the part of the search graph that has been explicitly generated so far is used rather than two lists, OPEN and CLOSED in previous algorithms.
- Each node in the graph will point both down to its immediate successors and up to its immediate predecessor.
- Each node in a graph will also have associated with it an h value (an estimate of the cost of a path from current node to a set of solution nodes).
- We will not store g (cost from start to current node) as it is not possible to compute a single such value since there may be many paths to the same state.
- Also such value is not necessary because of the topdown traversing of the best-known path which guarantees that only nodes that are on the best path will ever be considered for expansion. So h will be good estimate for AND/OR graph search.

# AO\* Algorithm (Example)



Unnecessary backward  
propagation

# AO\* Algorithm (Example)



# Constraints Satisfaction Problem (CSP)

- Standard search problem:
  - **state** is a "black box" – any data structure that supports successor function, heuristic function, and goal test
- **CSP:**
  - **state** is defined by **variables**  $X_i$  with **values** from **domain**  $D_i$
  - **goal test** is a set of **constraints** specifying allowable combinations of values for subsets of variables
- Simple example of a **formal representation language**
- Allows useful **general-purpose** algorithms with more power than standard search algorithms

# Example: Map-Coloring



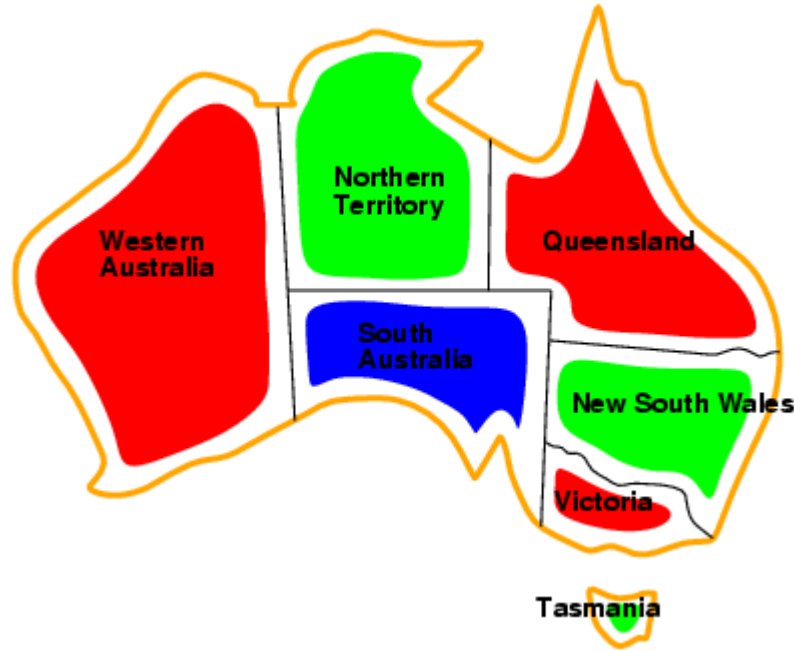
**Variables** WA, NT, Q, NSW, V, SA, T

**Domains**  $D_i = \{\text{red, green, blue}\}$

**Constraints:** adjacent regions must have different colors

e.g.,  $WA \neq NT$ , or  $(WA, NT) \in \{(\text{red, green}), (\text{red, blue}), (\text{green, red}), (\text{green, blue}), (\text{blue, red}), (\text{blue, green})\}$

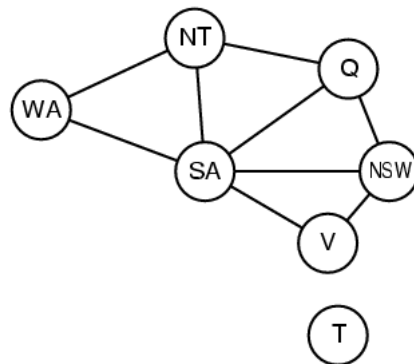
# Example: Map-Coloring



- Solutions are **complete** and **consistent** assignments satisfying all constraints,  
e.g., {WA = red, NT = green, Q = red, NSW = green, V = red, SA = blue, T = green}

# Constraint graph

- **Binary CSP:** each constraint relates at most two variables
- **Constraint graph:** nodes are variables, arcs show constraints



# Varieties of CSPs

- Discrete variables

- finite domains:

- n variables, domain size d  $\rightarrow O(d^n)$  complete assignments
- e.g., Boolean CSPs, incl.  $\sim$  Boolean satisfiability (NP-complete)

- infinite domains:

- integers, strings, etc.
- e.g., job scheduling, variables are start/end days for each job
- need a constraint language, e.g.,  $\text{StartJob}_1 + 5 \leq \text{StartJob}_3$

- Continuous variables

- e.g., start/end times for Hubble Telescope observations
- linear constraints solvable in poly time by LP methods

# Varieties of CSPs

- **Unary** constraints involve a single variable,  
SA  $\neq$  green
- **Binary** constraints involve pairs of variables,  
SA  $\neq$  WA
- Higher-order constraints involve 3 or more variables,  
cryptarithmic column constraints
- Preferences (soft constraints), e.g., **red** is better than  
**green**  
-often representable by a cost for each variable  
assignment  $\rightarrow$  constrained optimization problems

# Constraint Satisfaction

- As compared with a **straightforward search** procedure, viewing a problem as one of **constraint satisfaction** can reduce substantially the amount of search.

# Constraint Satisfaction

- Operates in a **space of constraint sets**.
- Initial state contains the **original constraints** given in the problem.
- A goal state is any state that has been **constrained “enough”**.

# Constraint Satisfaction

Two-step process:

1. Constraints are discovered and propagated as far as possible.
2. If there is still not a solution, then search begins, adding new constraints.

# Constraint Satisfaction

Two kinds of rules:

1. Rules that define valid constraint propagation.
2. Rules that suggest guesses when necessary.

# Constraint Satisfaction

- Many AI problems can be viewed as problems of **constraint satisfaction**.

Cryptarithmic puzzle:

$$\begin{array}{r} \text{SEND} \\ + \\ \hline \text{MORE} \\ \hline \text{MONEY} \end{array}$$

## Initial state:

- No two letters have the same value.
- The sum of the digits must be as shown.

Initial state of problem.

D=?

E=?

Y=?

N=?

R=?

O=?

S=?

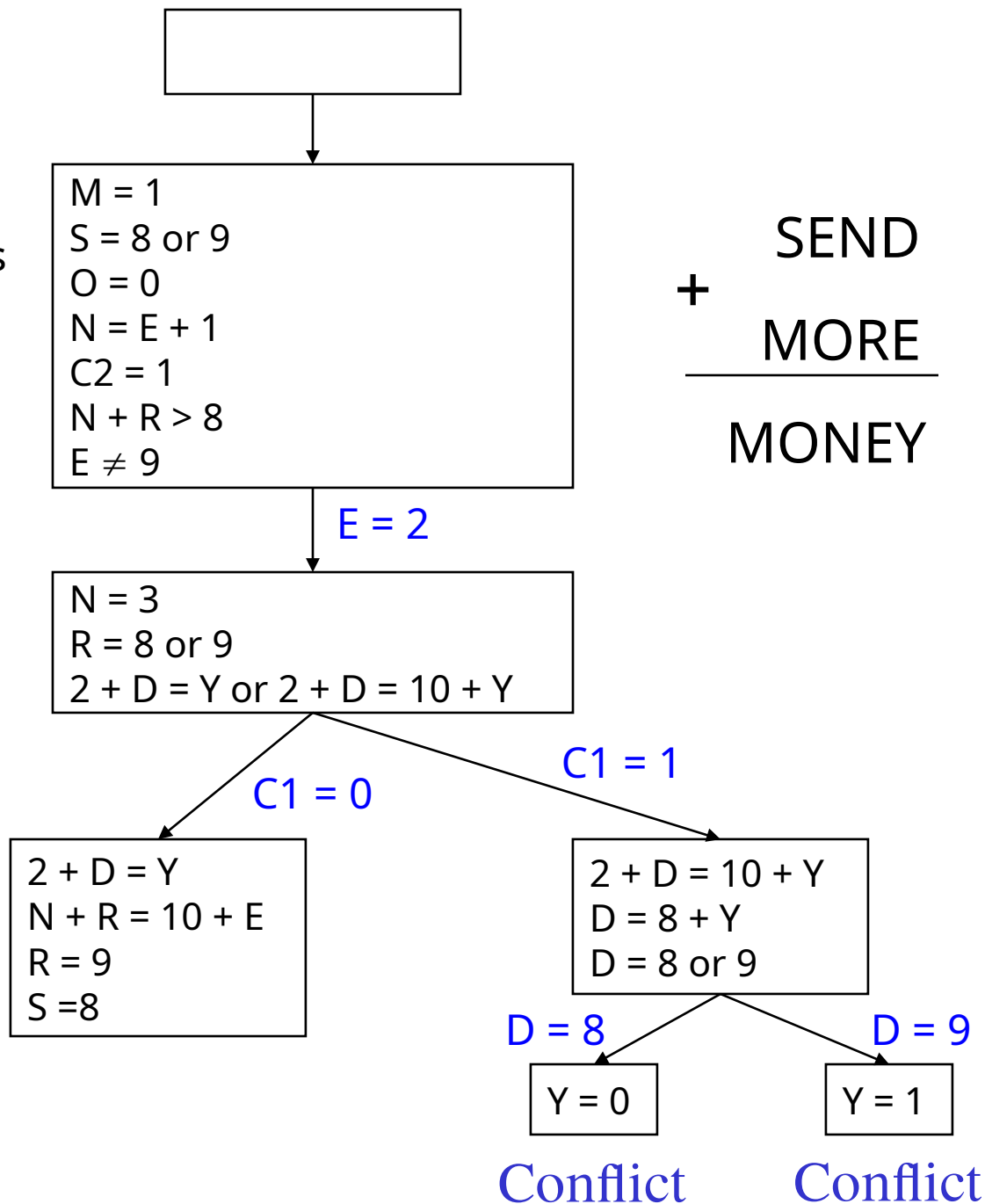
M=?

C1=?

C2=?

C1, C2, C3 stands for the carry variables respectively.

Goal State: the digits to the letters must be assigned in such a manner so that the sum is satisfied.



Step -1

M=1  
O=0

S=8 or 9

Let E=2

$$E(2) + O(0) + C2(1 \text{ or } 0) = N$$

If C2=1

If C2=0

$$E(2) + O(0) + C2(1) = N(3)$$

$$E(2) + O(0) + C2(0) = N(2)/x$$

Contradiction (Rule 3)

$$N(3) + R + C1(1 \text{ or } 0) = E(2)$$

If C1=1

If C1=0

R=8

R=9

S=9, C3=0

S=8, C3=1

$$D + E(2) = Y$$

D > 7 (to generate a carry)

X Contradiction (Rule 3)

$$D + E(2) = Y$$

D=4

D=5

D ≥ 7

X

Solution not Satisfied

(To Satisfy Y should generate carry)

Contradiction for value of 0 Comes

X

After Step 1 we derive are more conclusion that Y contradiction should generate a Carry. That is  $D+2 > 9$

Step - 2

$$\boxed{\begin{array}{c} M=1 \\ O=0 \end{array}}$$

Or  $S=8, S=9 \quad C3=1, C3=0$

Let  $E=3$

$$E(3)+O(0)+C2(1 \text{ or } 0)=M$$

$C2=1$

$C2=0$

$$E(3)+O(0)+C2(1)=N(4)$$

$$E(3)+O(0)+C2(0)=N(3)$$

X

Contradiction

$$N(4)+R+C1(1 \text{ or } 0)=E(3)$$

$C1=1$

$C1=0$

$R=8$

$S=9$

$$D+E(3)=y$$

Contraction (Y should generate carry in that case C1 cannot be equal do 0)

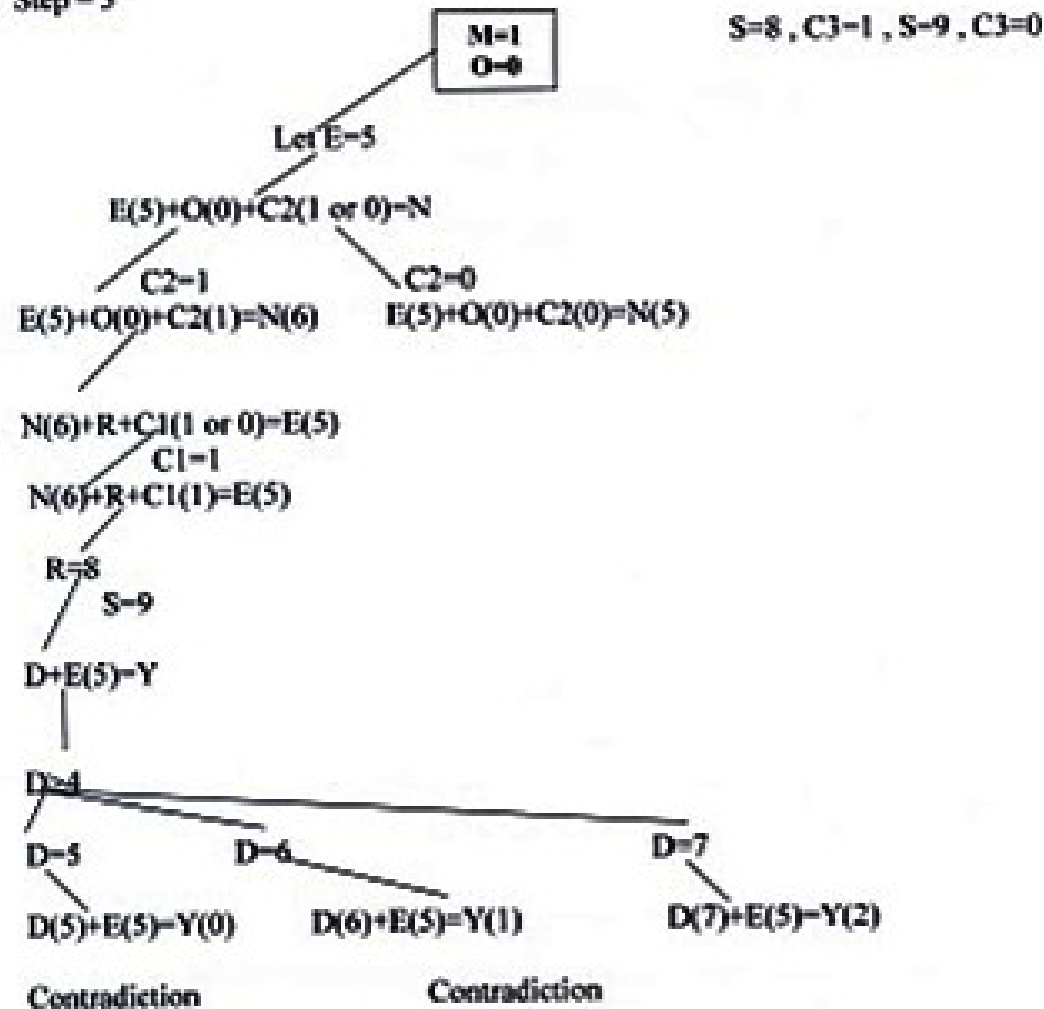
---

$D > 6$  (Contraction)

---

After Step 2 , we found that C1 cannot be Zero, Since Y has to generate a carry to satisfy goal state. From this step onwards, no need to branch for C1=0.

Step - 3



At Step (4) we have assigned a single digit to every letter in accordance with the constraints & production rules.

Now by backtracking , we find the different digits assigned to different letters and hence reach the solution state.

# Solution!!!

- **Y=2**
- **D=7**
- **S=9**
- **R=8**
- **N=6**
- **E=5**
- **O=0**
- **M=1**
- **C1=1**
- **C2=0**
- **C3=0**



$$\begin{array}{rcccc} & \mathbf{C3 (0)} & \mathbf{C2 (1)} & \mathbf{C1 (1)} & \\ & \mathbf{S (9)} & \mathbf{E (5)} & \mathbf{N (6)} & \mathbf{D (7)} \\ + & \mathbf{M (1)} & \mathbf{O (0)} & \mathbf{R (8)} & \mathbf{E (5)} \\ \hline & & & & \\ \hline \end{array}$$

$$\mathbf{M (1) O (0) N (6) E (5) Y (2)}$$

# Assignment

- Why not best-first search algorithm is not adequate for searching AND-OR graphs?

# More Cryptarithmic Problem

BLACK	7	9	2	0	8	
GREEN	5	3	4	4	6	
-----	-----	-----	-----	-----	-----	
ORANGE	1	3	2	6	5	4

CRASH	3	6	8	4	5	
HACKER	5	8	3	9	2	6
-----	-----	-----	-----	-----	-----	-----
REBOOT	6	2	0	7	7	1

CROSS	9	6	2	3	3	
ROADS	6	2	5	1	3	
-----	-----	-----	-----	-----	-----	
DANGER	1	5	8	7	4	6